Unit 6: Systems of Linear Equations and Inequalities

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Unit 6: Systems of Linear Equations and Inequalities

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Topic 1: Solving Systems of Linear Equations by Graphing

**Learning Objectives**
- Describe the creation and use of systems of equations.
- Graph a system of linear equations on the coordinate plane and identify its solution.

Topic 2: Solving Systems of Linear Equations by Substitution

**Learning Objectives**
- Define the substitution method.
- Use substitution to solve systems of equations.

Topic 3: Solving Systems of Linear Equations by Elimination

**Learning Objectives**
- Define the elimination method of solving systems of equations.
- Combine equations in a system to eliminate variables.

Lesson 2: Applying Systems of Equations

Topic 1: Rate Problems

**Learning Objectives**
- Use systems of equations to describe and solve problems involving rates.

Topic 2: Mixture problems

**Learning Objectives**
- Explain how mixtures are a type of rate.
- Use systems of equations to describe and solve problems involving mixtures.

Lesson 3: Graphing Systems of Inequalities

Topic 1: Graphing Systems of Inequalities

**Learning Objectives**
- Represent systems of linear inequalities as regions on the coordinate plane.
- Identify the bounded region for a system of inequalities.
- Determine if a given point is a solution of a system of inequalities.
# Unit 6 - Media Run Times

## Lesson 1
- Topic 1, Presentation – 3.3 minutes
- Topic 1, Worked Example 1 – 3 minutes
- Topic 1, Worked Example 2 – 3.4 minutes
- Topic 1, Worked Example 3 – 5 minutes

- Topic 2, Presentation – 4.4 minutes
- Topic 2, Worked Example 1 – 3.7 minutes
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- Topic 3, Presentation – 4.5 minutes
- Topic 3, Worked Example 1 – 5.8 minutes
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- Topic 3, Worked Example 3 – 4.9 minutes

## Lesson 2
- Topic 1, Presentation – 3.9 minutes
- Topic 1, Worked Example 1 – 5.9 minutes
- Topic 1, Worked Example 2 – 10 minutes
- Topic 1, Worked Example 3 – 4.9 minutes

- Topic 2, Presentation – 3.8 minutes
- Topic 2, Worked Example 1 – 2.7 minutes
- Topic 2, Worked Example 2 – 6.7 minutes
- Topic 2, Worked Example 3 – 4 minutes

## Lesson 3
- Topic 1, Presentation – 6.4 minutes
- Topic 1, Worked Example 1 – 4.9 minutes
- Topic 1, Worked Example 2 – 3.3 minutes
- Topic 1, Worked Example 3 – 9.7 minutes
Unit 6: Systems of Linear Equations and Inequalities

Instructor Notes

The Mathematics of Linear Systems

Unit 6 extends the skills learned in Units 4 and 5 (analyzing individual equations and inequalities) into working with linear systems. Students will learn three techniques for solving systems - graphing, substitution, and elimination. They'll also see how to apply these methods to writing and solving rate and mixture problems.

Although students are quite familiar with algebraic statements by this point in the course, understanding and solving systems of multiple equations and inequalities can still be difficult.

Teaching Tips: Conceptual Challenges and Approaches

Up to this point for students, the word 'solution' has usually meant a single number or a range of numbers: this has been true for them since 2nd grade when they learned to find the missing □ in a number sentence like 4 + □ = 10. Now they're being asked for a solution that is a coordinate (x, y). This will be a conceptual hurdle for many students. We start the unit with a topic on finding solutions to systems of equations by graphing in order to help build an intuitive feel for the concept of a linear system. Actually seeing two (or more) lines on a plane conveys the idea of the solution of a system more clearly than words or symbols. This will also help students understand the three possible scenarios for the solutions of linear systems:

<table>
<thead>
<tr>
<th>One Solution</th>
<th>No Solutions</th>
<th>Infinite Solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="One Solution Diagram" /></td>
<td><img src="image2.png" alt="No Solutions Diagram" /></td>
<td><img src="image3.png" alt="Infinite Solutions Diagram" /></td>
</tr>
</tbody>
</table>

If the graphs of the equations intersect, then there is one solution that is true for both equations. That solution is a point.

If the graphs of the equations do not intersect (they are parallel), then there are no solutions that are true for both equations.

If the graphs of the equations are the same, then there are an infinite number of solutions (all of them points) that are true for both equations.
It is important that students graph examples of all three types of systems and discuss their solutions.

**Teaching Tips: Algorithmic Challenges and Approaches**

Most students can easily look at the graph of a system and see if the lines cross or cover one another. Many students will also be able to understand and use substitution as a method for solving a system of linear equations. The idea is fairly easy to grasp, and once the substitution is performed, all they have to do is follow the familiar steps for solving equations and inequalities.

But students are likely to struggle with elimination (or linear combination) as a method for solving linear systems. We suggest teaching this technique last after graphing (easiest to understand), and substitution (most familiar mathematics). The method of linear combination forms the basis of a lot of later mathematics (matrices, vectors, linear programming etc.) To ease them into the idea, students may find it helpful to see an example of linear combination at work.

**Example**

```
RGB Color Demo

Red 0.500
Green 0.500
Blue 0.500

Color = 0.500 [1 0 0] + 0.500 [0 1 0] + 0.500 [0 0 1]

Color as a linear combination.

http://mathdemos.gcsu.edu/mathdemos/rgb-demo1/
```

The color in the box is produced by a linear combination of three colors. Moving a slider multiplies each color vector (Red, Green, and Blue) by a different numerical factor to produce a different linear combination. **Note:** This is NOT a direct example of elimination, but it nicely illustrates a system of equations, and provides a real world application that uses similar mathematics.

When using the algorithm for solving by elimination there are two specific areas students will likely find difficult, choosing factors and keeping track of the changing equations. Choosing appropriate factors to multiply the equations takes both intuition and practice. First introduce students to the method and give them some time, probably several days, to work with it and solve problems. You may have to give students procedures they can follow by rote for a while. Once they have gained some familiarity with the technique, take time to make sure they know why they are performing this procedure.
Example
Place students in small groups and have each group produce a description of the method as if explaining it to a new student in the class.
Students need to be able to produce an explanation along the lines of:
“We’re trying to find a way to add or subtract the two equations to eliminate one of the variable terms. We look for terms that when multiplied or divided by any number would be the same or the opposite of the similar term in the other equation. Once we’ve eliminated one term we can easily find a value for the other, and then find x and y.”

A second stumbling block is keeping track of what happens to equations as they are manipulated to allow variables to be eliminated and substituted. Students will have a much easier time keeping things straight if they’re encouraged to label their work as they go. The following layout may be helpful:

<table>
<thead>
<tr>
<th>Operations</th>
<th>Equations</th>
<th>Labels</th>
</tr>
</thead>
<tbody>
<tr>
<td>multiply equation 1 by -3</td>
<td>$-6x + 9y = -24$</td>
<td>new equation 1</td>
</tr>
<tr>
<td>multiply equation 2 by 2</td>
<td>$6x + 8y = 92$</td>
<td>new equation 2</td>
</tr>
<tr>
<td>add new equations</td>
<td>$-6x + 9y = -24$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$6x + 8y = 92$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$17y = 68$</td>
<td></td>
</tr>
<tr>
<td>simplify</td>
<td>$y = 4$</td>
<td></td>
</tr>
<tr>
<td>substitute value of y into original equation 1</td>
<td>$2x - 3(4) = 8$</td>
<td></td>
</tr>
<tr>
<td>solve for x</td>
<td>$x = 10$</td>
<td></td>
</tr>
<tr>
<td>write x and y as co-ordinates</td>
<td>$(x, y) = (10, 4)$</td>
<td>Solution to the System</td>
</tr>
</tbody>
</table>

Helping students keep track of the details of this more complicated procedure will assist them in retaining a “big picture” view of what they are trying to achieve.
Summary

This unit teaches students how to solve systems of linear equations using graphing, substitutions, and elimination. They'll also learn how to represent systems of inequalities on the coordinate plane. Finally, they'll practice applying these techniques to analyzing rate and mixture problems. To help students understand the idea of systems, we suggest beginning with graphing. To help them become skilled at substitution and elimination, show them how to label and organize problems and make sure they get plenty of practice.
Unit 6: Systems of Linear Equations and Inequalities

Instructor Overview
Tutor Simulation: Mixing Punch

Purpose
This simulation is designed to challenge a student’s understanding of systems of equations. Students will be asked to apply what they have learned to solve a real world problem by demonstrating understanding of the following areas:

- Patterns
- Systems of Equations
- Percentages and Ratios
- Writing Equations and Systems
- Proportional Reasoning
- Solving Equations and Systems

Problem
Students are given the following problem:

Your friend Angelo invited you to a party, but before you start dancing the night away, Angelo has asked you to help him out with mixing the punch. He has a recipe, but needs to modify it for a larger crowd. Your challenge will be to help Angelo figure out the proper proportions to adapt the punch recipe for more people.

Recommendations
Tutor simulations are designed to give students a chance to assess their understanding of unit material in a personal, risk-free situation. Before directing students to the simulation,

- make sure they have completed all other unit material.
- explain the mechanics of tutor simulations
  - Students will be given a problem and then guided through its solution by a video tutor;
  - After each answer is chosen, students should wait for tutor feedback before continuing;
  - After the simulation is completed, students will be given an assessment of their efforts. If areas of concern are found, the students should review unit materials or seek help from their instructor.
- emphasize that this is an exploration, not an exam.
Unit 6: Systems of Linear Equations and Inequalities

Instructor Overview
Puzzle: Apples and Oranges

Objective

Apples and Oranges is a puzzle that represents systems of equations visually by grouping objects in various combinations. To solve the puzzles, students must translate the images into two or more equations and then use substitution or elimination to find the value of each variable. Students will need to understand how to write and analyze systems of equations to succeed.

Figure 1. What’s More? uses fruits and other objects to test a student's grasp of techniques for solving systems of equations.

Description

This puzzle has three levels of difficulty. In each level, multiple objects are arranged in various combinations. Players must write an equation that represents each grouping, and then use the resulting system of equations to solve for each variable. When the learner finds the right answer, the algebraic equations underlying the problem and the solution are shown.

At the easy level there are three kinds of objects (apples, oranges, and cherries) grouped into two arrangements. The medium level also presents a two equation system,
but the three objects (opals, emeralds, and rubies) are grouped in a more complicated way. At the hard level, there are four objects (rockets, spaceships, ray guns, and asteroids) arranged into a system of three equations.

Players score points for solving the puzzles and the points accumulate across puzzles. The ten puzzles at each level are scripted, but they are sufficiently difficult to solve visually that there is opportunity for replay. Apples and Oranges is suitable for both individual play and group learning in a classroom setting.
Unit 6: Systems of Linear Equations and Inequalities

Instructor Overview
Project: Say What?

Student Instructions

Introduction
Traditionally, in algebra coursework, you are given a problem situation and asked to create an equation or system of equations to solve the problem. You have no doubt encountered many problems asking, “How many quarters did Bobby have?” or “How much older than Grace is Juan?” Although solving these types of problems is a core component of becoming proficient at algebra, beginning to think about the linear relationships that are found in the real world allows you to extend and apply your algebraic knowledge. In this activity, you will discover that a line on a coordinate plane can have many meanings, depending on the labels associated with the axes.

Task
Working together with your group, you will develop your own storyline to represent two linear relationships. Creativity will be the key in this project, as you get to define your own independent and dependent variables and then describe what the solution to your system would represent within the confines of your storyline. After developing your own linear relationships and storyline, you will then be put to the test as you try to interpret another group’s graph. Get ready to start thinking outside the box!

Instructions
Solve each problem in the order given. Save your work along the way, as you will create a display at the conclusion of the project.

1 First problem:
Choose one of the following sets of domain and range values. You will use the domain and range values to create your graph, equations, and storyline.

   Domain: 0 ≤ x ≤ 50    Domain: 0 ≤ x ≤ 40
   Range: 0 ≤ y ≤ 400    Range: 0 ≤ y ≤ 100

   Domain: 0 ≤ x ≤ 10     Domain: 0 ≤ x ≤ 1000
   Range: 0 ≤ y ≤ 800     Range: 0 ≤ y ≤ 2000

   Domain: 0 ≤ x ≤ 20     Domain: 0 ≤ x ≤ 500
   Range: 0 ≤ y ≤ 300     Range: 0 ≤ y ≤ 1000

2 Second problem:
Considering the domain and range values that you chose above. Begin discussing what your possible independent and dependent variables might be. Remember, creativity makes your project stand out! Some examples include:

<table>
<thead>
<tr>
<th>Independent</th>
<th>Dependent</th>
</tr>
</thead>
<tbody>
<tr>
<td>time</td>
<td>distance</td>
</tr>
<tr>
<td>number of students</td>
<td>number of pizzas</td>
</tr>
<tr>
<td>age</td>
<td>height</td>
</tr>
</tbody>
</table>

Once you have decided on your variables, begin thinking about your storyline. You will write a more detailed storyline later. For now, simply consider what each line will represent based on your variables. You will have two lines on your graph. The first line will represent your first scenario and the second line will represent the second scenario. Make a data table and sketch a graph of each line. You will make a more professional looking graph next.

3 Third Problem:
GeoGebra can be used to make the graphs for this project. GeoGebra is a free program. The download can be found at: http://www.geogebra.org/cms/en/download

To create your graph, choose the Drawing Pad under the Options menu. You will now be able to enter your minimum and maximum values for each axis, as well as units and labels. While still in the Drawing Pad, select the grid tab and check grid to show gridlines on your graph.

(Hint: You will need to set a minimum a little less than zero for both your domain and range. If you do not set the minimum a little less than zero, the axes are not visible on your graph. You will need to play around with your minimums until you are happy with your axes.)

Once your graph is defined, you will need to plot your points. In order to plot a point, select the “New Point” button and then click on the grid to drop the point. Once your points are defined, select the “Line Through Two Points” button. Click on any two points to create a line.

You will now need to save or print two copies of the file. One will have the labels and units defined. The other will have the units and labels removed.

(Hint: To remove the labels and units, go to the Drawing Pad. The graph with no labels and units will be given to another team.)
4 Fourth problem:
Using the points defined on your graph, find the equation of each line. Now, solve the system using substitution or elimination. Show your work neatly, as it will be part of your display. Compare your solution to the graph to ensure they match. If not, check your algebra.

(Hint: To find the equation of a line given two ordered pairs, you first need to find the slope. Then use either point-slope form or slope-intercept form to find the equation.)

5 Fifth problem:
Write a detailed storyline to describe each line and the solution to the system. Be sure to include a backdrop for your story line. Keep your story believable, yet creative. Be sure to describe the story associated with each line and then describe what the solution to the system means within the context of your storyline.

Collaboration
Trade unlabeled graphs with another group. With your new graph, again find the equation of each line. Use substitution or elimination to find the solution to the system. Check to ensure that the solution matches with the graph. Now, create a new detailed storyline. This detailed storyline will be part of the other team’s project. Do your best and be creative!

Conclusions
Create a poster display including your labeled graph, algebraic solution to the system, and detailed storyline. Then, collect the second storyline created by the other team. Include it on your display, as well. In order to prepare a more professional looking product, consider using Microsoft Word or a similar word processing program to type your detailed storyline. Including hand-drawn or computer-generated graphics will also enhance your display.

As an alternative to a poster, consider creating a website for your project. You can save your GeoGebra graph and import it in HTML or as a picture. An easy, free place to create a website is www.weebly.com. If you choose to create a website, be sure to include your labeled graph, algebraic solution to the system, detailed storyline, and the other group’s storyline that corresponds to your graph. Also, consider inserting a page for a poll to determine which storyline is most believable and which is most creative.

Instructor Notes

Assignment Procedures

Students are asked to create graphs and embed them in their project reports. We suggest using the free program GeoGebra for this, and the instructions provide some guidance on how to use it. However, it would be helpful to spend a little time as a class explaining how this (or another preferred graphing program) works.

In the fifth problem, students are asked to create a storyline around their data set. Students are much more familiar with pulling information out of a word problem than they
are with making up a scenario to describe a set of values. It may be helpful to provide an example, such as the following:

**Storyline:** Mr. Barnum is putting on a big-top circus event. For his center ring act, he needs two elephants of the exact same weight. Bucky, the larger elephant, currently weighs in at 5000 kg. Sally, the smaller elephant, currently weighs in at 3000 kg. In consultation with his veterinarian, Mr. Barnum has put Bucky on a restrictive diet and is giving Sally a high-calorie diet rich in elephant treats. Line AB represents Bucky’s weight over time and Line CD represents Sally’s weight over time. If the elephants follow their special diets exactly, Mr. Barnum knows that the two elephants will be the same weight right around day 64 when they will each weigh roughly 4300 kg.

(The exact answers using algebra will be worked out and shown on the final project.)

**Recommendations:**

- have students work in teams to encourage brainstorming and cooperative learning.
- assign a specific timeline for completion of the project that includes milestone dates.
- provide students feedback as they complete each milestone.
- ensure that each member of student groups has a specific job.

**Technology Integration**

This project provides abundant opportunities for technology integration, and gives students the chance to research and collaborate using online technology.

Students can neatly type their algebraic equations and solutions directly in Microsoft Power Point by choosing Object under the Insert menu. They can then work in Equation Editor to type the algebra neatly on each slide. It takes some practice to decipher the
buttons on the Equation Editor menu. A quick mini-lesson on Equation Editor would allow the students to create a professional looking Power Point with integrated algebra.

The following are examples of free internet resources that can be used to support this project:

http://www.moodle.org
An Open Source Course Management System (CMS), also known as a Learning Management System (LMS) or a Virtual Learning Environment (VLE). Moodle has become very popular among educators around the world as a tool for creating online dynamic websites for their students.

http://www.wikispaces.com/site/for/teachers or http://pbworks.com/content/edu+overview
Lets you create a secure online Wiki workspace in about 60 seconds. Encourage classroom participation with interactive Wiki pages that students can view and edit from any computer. Share class resources and completed student work with parents.

http://www.docs.google.com
Allows students to collaborate in real-time from any computer. Google Docs provides free access and storage for word processing, spreadsheets, presentations, and surveys. This is ideal for group projects.

http://why.openoffice.org/
The leading open-source office software suite for word processing, spreadsheets, presentations, graphics, databases and more. It can read and write files from other common office software packages like Microsoft Word or Excel and MacWorks. It can be downloaded and used completely free of charge for any purpose.
## Rubric

<table>
<thead>
<tr>
<th>Score</th>
<th>Content</th>
<th>Presentation</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Your project appropriately answers each of the problems. Your algebraic equations are set up properly. The step-by-step solution to each equation is given. Your project correctly identifies which vendor is the better choice for the party and justifies the decision mathematically.</td>
<td>Your project contains information presented in a logical and interesting sequence that is easy to follow. Your project is professional looking with graphics and effective use of color.</td>
</tr>
<tr>
<td>3</td>
<td>Your project answers each of the problems. Your algebraic equations are set up and step-by-step solutions are given. Minor errors may be noted. Your project identifies which vendor is the better choice for the party and justifies the decision mathematically. Minor errors may be noted.</td>
<td>Your project contains information presented in a logical sequence that is easy to follow. Your project is neat with graphics and effective use of color.</td>
</tr>
<tr>
<td>2</td>
<td>Your project attempts to answer each of the problems. Algebraic equations are attempted and solutions are given. Major errors are noted. Your project attempts to identify which vendor is the better choice for the party and justifies the decision mathematically. Major errors are noted.</td>
<td>Your project is hard to follow because the material is presented in a manner that jumps around between unconnected topics. Your project contains low quality graphics and colors that do not add interest to the project.</td>
</tr>
</tbody>
</table>
## Unit 6: Algebra - Systems of Linear Equations and Inequalities

### Glossary

| **boundary line** | a line that represents the edge of a linear inequality; if points along the boundary line are included in the solution set, then a solid line is used; if points along the boundary line are not included in the solution set, then a dashed line is used |
| **bounded region** | the set of solutions that are true for all of the linear inequalities under consideration |
| **elimination method** | a method of solving a system of equations by adding or subtracting equations in order to eliminate a common variable |
| **half-plane** | on a coordinate plane, the shape of the region of possible solutions generated by a single inequality |
| **rate** | a mathematical way of relating two quantities, which usually are measured in different units |
| **substitution method** | a method of solving a system of equations by substituting one quantity in for an equivalent quantity |
| **system of equations** | a set of two or more equations that share two or more unknowns |
| **system of inequalities** | a set of two or more inequalities that must hold true at the same time |
# NROC Algebra 1--An Open Course
## Unit 6
Mapped to Common Core State Standards, Mathematics

<table>
<thead>
<tr>
<th>Algebra 1</th>
<th>Systems of Linear Equations and Inequalities</th>
<th>Solving Systems of Linear Equations</th>
<th>Solving Systems of Linear Equations by Graphing</th>
</tr>
</thead>
</table>

### Grade: 7 - Adopted 2010

**STRAND / DOMAIN** | **CC.7.EE.** | Expressions and Equations  
**CATEGORY / CLUSTER** | Use properties of operations to generate equivalent expressions.  
**STANDARD** | 7.EE.2. | Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example, $a + 0.05a = 1.05a$ means that “increase by 5%” is the same as “multiply by 1.05.”

### Grade: 8 - Adopted 2010

**STRAND / DOMAIN** | **CC.8.EE.** | Expressions and Equations  
**CATEGORY / CLUSTER** | Analyze and solve linear equations and pairs of simultaneous linear equations.  
**STANDARD** | 8.EE.8. | Analyze and solve pairs of simultaneous linear equations.  
**EXPECTATION** | 8.EE.8.a. | Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.  
**EXPECTATION** | 8.EE.8.b. | Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.  
**EXPECTATION** | 8.EE.8.c. | Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

### Grade: 9-12 - Adopted 2010

**STRAND / DOMAIN** | **CC.A.** | Algebra  
**CATEGORY / CLUSTER** | A-CED. | Creating Equations  
**STANDARD** | Create equations that describe numbers or relationships.  
**EXPECTATION** | A-CED.3. | Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.

**STRAND / DOMAIN** | **CC.A.** | Algebra  
**CATEGORY / CLUSTER** | A-REI. | Reasoning with Equations and Inequalities  
**STANDARD** | Solve systems of equations.  
**EXPECTATION** | A-REI.6. | Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

| 6.18 |
### Algebra 1 | Systems of Linear Equations and Inequalities | Solving Systems of Linear Equations by Substitution

<table>
<thead>
<tr>
<th>CATEGORY / CLUSTER</th>
<th>A-REI.</th>
<th>Reasoning with Equations and Inequalities</th>
</tr>
</thead>
<tbody>
<tr>
<td>STANDARD</td>
<td></td>
<td>Represent and solve equations and inequalities graphically.</td>
</tr>
<tr>
<td>EXPECTATION</td>
<td>A-REI.11.</td>
<td>Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.</td>
</tr>
</tbody>
</table>

Grade: 8 - Adopted 2010

<table>
<thead>
<tr>
<th>STRAND / DOMAIN</th>
<th>CC.8.EE.</th>
<th>Expressions and Equations</th>
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<tbody>
<tr>
<td>CATEGORY / CLUSTER</td>
<td></td>
<td>Analyze and solve linear equations and pairs of simultaneous linear equations.</td>
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<tr>
<td>EXPECTATION</td>
<td>8.EE.8.b.</td>
<td>Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.</td>
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Grade: 9-12 - Adopted 2010

<table>
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<tr>
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<th>Algebra</th>
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<tr>
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<td>Reasoning with Equations and Inequalities</td>
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<td>Analyze and solve linear equations and pairs of simultaneous linear equations.</td>
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<td>EXPECTATION</td>
<td>8.EE.8.b.</td>
<td>Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.</td>
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Grade: 9-12 - Adopted 2010

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<tr>
<td>CATEGORY / CLUSTER</td>
<td>A-REI.</td>
<td>Reasoning with Equations and Inequalities</td>
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<tr>
<td>STANDARD</td>
<td></td>
<td>Solve systems of equations.</td>
</tr>
<tr>
<td>EXPECTATION</td>
<td>A-REI.5.</td>
<td>Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.</td>
</tr>
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</table>
### Algebra 1 | Systems of Linear Equations and Inequalities | Applying Systems of Equations | Rate Problems

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### Grade: 8 - Adopted 2010

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<th>Expressions and Equations</th>
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<tbody>
<tr>
<td><strong>CATEGORY / CLUSTER</strong></td>
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<td>Analyze and solve linear equations and pairs of simultaneous linear equations.</td>
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<tr>
<td><strong>EXPECTATION</strong></td>
<td>A-REI.6.</td>
<td>Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.</td>
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### Algebra 1 | Systems of Linear Equations and Inequalities | Applying Systems of Equations | Mixture Problems

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### Algebra 1—An Open Course

#### Professional Development

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<td>EXPECTATION</td>
<td>8.EE.8.c.</td>
<td>Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.</td>
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<tr>
<td>CATEGORY / CLUSTER</td>
<td>A-CED.</td>
<td>Creating Equations</td>
</tr>
<tr>
<td>STANDARD</td>
<td></td>
<td>Create equations that describe numbers or relationships.</td>
</tr>
<tr>
<td>EXPECTATION</td>
<td>A-CED.3.</td>
<td>Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.</td>
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Algebra 1 | Systems of Linear Equations and Inequalities | Graphing Systems of Inequalities | Graphing Systems of Inequalities

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<td>Represent and solve equations and inequalities graphically.</td>
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<td>A-REI.12.</td>
<td>Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.</td>
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