

Developmental Math – An Open Curriculum Instructor Guide

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Unit 10: Solving Equations and Inequalities

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Instructor Guide

Unit 10 – Learning Objectives

Unit 10: Solving Equations and Inequalities

Lesson 1: Solving Equations

Topic 1: Solving One-Step Equations Using Properties of Equality

Learning Objectives

- Solve algebraic equations using the addition property of equality.
- Solve algebraic equations using the multiplication property of equality.

Topic 2: Solving Multi-Step Equations

Learning Objectives

- Use properties of equality together to isolate variables and solve algebraic equations.
- Use the properties of equality and the distributive property to solve equations containing parentheses.

Topic 3: Special Cases and Applications

Learning Objectives

- Solve equations that have one solution, no solution, or an infinite number of solutions.
- Solve application problems by using an equation in one variable.

Topic 4: Formulas

Learning Objectives

- Evaluate a formula using substitution.
- Rearrange formulas to isolate specific variables.

Lesson 2: Solving Inequalities

Topic 1: Solving One-Step Inequalities

Learning Objectives

- Represent inequalities on a number line.
- Use the addition property of inequality to isolate variables and solve algebraic inequalities, and express their solutions graphically.
- Use the multiplication property of inequality to isolate variables and solve algebraic inequalities, and express their solutions graphically.

Topic 2: Multi-Step Inequalities

Learning Objectives

- Use the properties of inequality together to isolate variables and solve algebraic inequalities, and express their solutions graphically.
- Simplify and solve algebraic inequalities using the distributive property to clear parentheses and fractions.

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Lesson 3: Compound Inequalities and Absolute Value

Topic 1: Compound Inequalities

Learning Objectives

- Solve compound inequalities in the form of "or" and express the solution graphically.
- Solve compound inequalities in the form of "and" and express the solution graphically.
- Solve compound inequalities in the form $a < x < b$.
- Identify cases with no solution.

Topic 2: Equations and Inequalities and Absolute Value

Learning Objectives

- Solve equations containing absolute values.
- Solve inequalities containing absolute values.
- Identify cases of equations and inequalities containing absolute values which have no solutions.

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Unit 10 – Instructor Notes

Unit 10: Solving Equations and Inequalities

Instructor Notes

The Mathematics of Solving Equations and Inequalities

Most students taking algebra already know the techniques for solving simple equations. This unit explores the principles and properties used to solve multi-step equations. It covers the parts, simplification, rearrangement, and solution of both linear equations and inequalities. In addition, it introduces compound inequalities and absolute value equations to intermediate algebra students.

The course work emphasizes understanding the properties of equality and inequality and the distributive property. Students must be able to apply these concepts in order to succeed in developmental math and algebra. In addition to solving equations and inequalities, students will also learn how to translate word problems into algebraic equations and inequalities.

Teaching Tips: Challenges and Approaches

Variables and expressions were covered in Unit 9: Real Numbers. In this unit, students are introduced to algebraic equations and inequalities. For many of them, this is where math gets both scary and frustrating. As they begin to test these deeper waters, make sure they have a thorough grounding in the meaning of common mathematical words and symbols, the properties of numbers, and in basic problem-solving strategies. As always, we recommend starting with simple, perhaps even review, problems to illustrate key concepts. In particular, make sure your students understand the properties that are used to solve equations and inequalities. This will help them realize that problem solving in algebra isn't a mystery but a series of logical steps that will always work.

Common Mistakes

As the mathematics in this course becomes more complicated, it gets increasingly more important that students understand exactly what math words and symbols mean. Although they've seen and used symbols like the equals sign, absolute value bars, and greater than/less than signs before, they may not fully grasp the details. Review all definitions thoroughly before presenting any problems. Rather than let students make basic mistakes and have to correct them later, it may be useful to run through some of the more common errors as a classroom exercise before students internalize the misunderstandings.

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Equality Issues

Remind your students that an equation is a mathematical statement of two equivalent expressions joined by an equals sign. Sometimes students will try to solve “ $x + 6$ ” thinking it is “ $x + 6 = 0$ ”.

Misunderstanding the equals sign can make it difficult for students to maintain the equality of an equation. When solving an equation such as $6 + 3x + 2 = 4x + 3$, students realize that they need to subtract 2, but often do so from all the constants rather than from each side of the equation. It may seem like the best cure for this is simply to insist they memorize some problem solving procedures, but a more valuable approach is to improve their understanding of the equals sign. Provide visual and/or hands on analogies like comparing the sides of an equation to the arms of a balance scale, and students will have a stronger feel for what 'balance' and 'equality' really mean in math.

Parentheses Problems

Many students fail to treat expressions in parentheses as a unit. For example, when solving the equation $3(x + 5) = 24$, quite a few will forget to distribute 3 to both terms inside the parentheses, while others will begin by subtracting 5 from both sides of the equation.

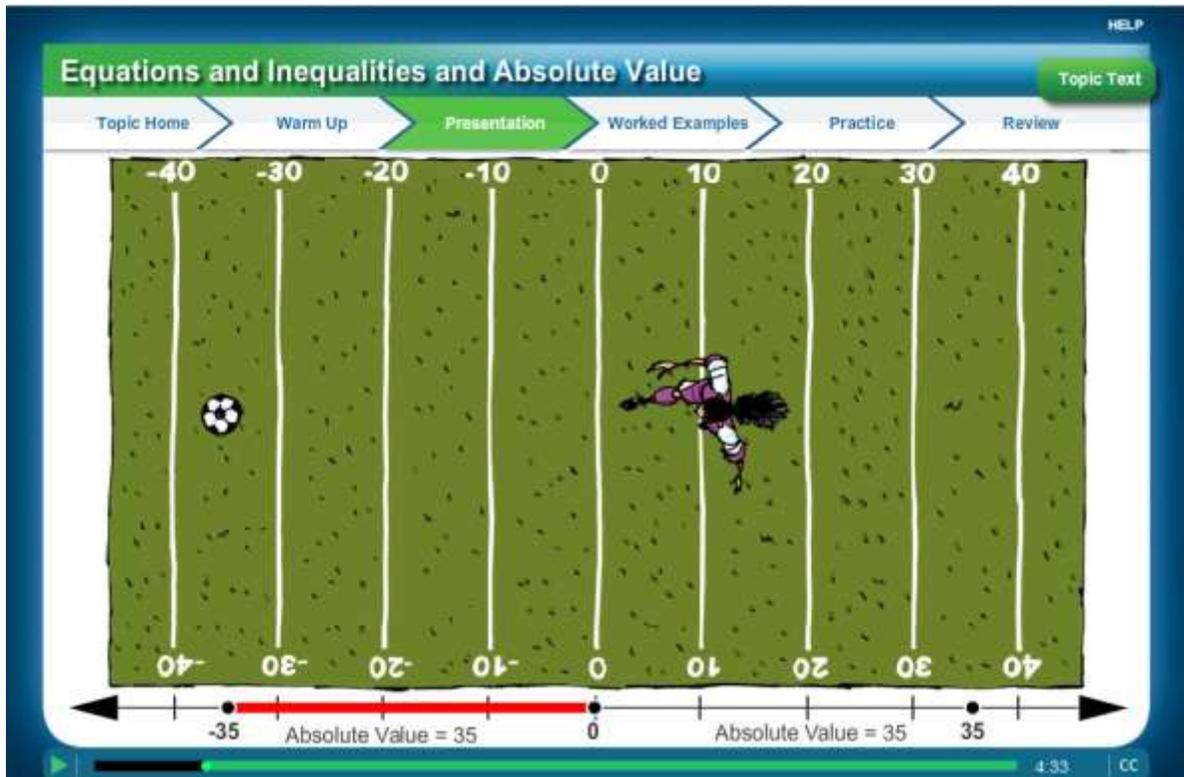
Flipping Inequalities

Once students are comfortable solving equations, solving inequalities is fairly straightforward, except when it comes to operations with negative numbers. Some students will forget that when multiplying or dividing an inequality by a negative number, the inequality sign must be reversed. Others will do that correctly, but also flip the direction of the inequality if they subtract or add a negative. Instead of just laying out the rules, be sure to illustrate why the sign must be reversed for some operations but not others. Work through some problems, and diagram the answers on a number line as well. Chose a simple inequality, such as $6 > 5$, then multiply it by -1 and show that the sign must be flipped to keep the relationship true. Then add and subtract a negative number, and compare what happens to the inequality.

Absolute Confusion

When students see an absolute value, they'll often respond by simply changing all negative signs in the vicinity to positive ones. Some will clear the absolute value bars by treating them like parentheses and using the distributive property. They may try finding the opposite of just part of the expression inside absolute value bars, rather than the entire quantity. (For example, rewriting $|2x - 5| = 11$ as “ $2x - 5 = 11$ or $2x + 5 = 11$.”) Many students will get so caught up in problem-solving that they forget that an absolute value can never be negative. They'll dive right in to an equation like $|x - 4| = -3$ or an inequality like $|x - 4| < -3$, rewriting these as $-3 < x - 4 < 3$ and get the reasonable-looking answer of $1 < x < 7$. Show them that these procedures are wrong by testing the results in the original equations. Then be sure to discuss why they are wrong as well—try using number lines and real world comparisons to help them understand that absolute value describes distance without direction. Here's an example:

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[From Lesson 3, Topic 2, Presentation]

Ask a student how far she can kick a soccer ball and she'll tell you in terms of so many yards—the answer is a type of absolute value, where distance is the only value that matters, and direction is irrelevant.

Problem-solving Techniques

Most developmental math students will find it easier to keep track of operations if they work underneath problems, as seen in the example below:

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Example	
Problem	Solve $x + 7 = 42$.
$x + 7 = 42$	Since 7 is being added to the variable, subtract 7 to isolate the variable.
$ \begin{array}{r} x + 7 = 42 \\ -7 \quad -7 \\ \hline x + 0 = 35 \end{array} $	To keep the equation balanced, subtract 7 from both sides of the equation.
Answer	$x = 35$

[From Lesson 1, Topic 1, Topic Text]

Although some students might be used to working in line with an equation ($x + 7 - 7 = 42 - 7$), this tends to lead to more errors and should be discouraged.

Translating word problems into equations is often quite challenging. Word problems can be especially tricky for students who speak English as a second language, but many English-only speakers are equally perplexed. Point out that when students read a problem carefully, they'll find a statement or question that previews the form their answer should take. (Hint: It's usually at the end.) If they write this answer statement first, with blanks for the values, they'll know what variables to solve for. Consider this problem:

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Example	
Problem Amanda's dad is twice as old as she is today. The sum of their ages is 66. Find the ages of Amanda and her dad.	
We need to find Amanda's age and her father's age.	<i>What is the problem asking?</i>
Let a = Amanda's age	Assign a variable to the unknown.
$2a$ = Father's age	The father's age is two times Amanda's age.
$a + 2a = 66$	Amanda's age added to her father's age is equal to 66.
$a + 2a = 66$	Solve the equation for the variable.
$\frac{3a}{3} = \frac{66}{3}$	
$a = 22$	
Amanda's age = 22 Father's age = $2a = 2(22) = 44$	Use Amanda's age to find her father's age.
Amanda's father's age is double Amanda's age and their sum is 66. The solutions make sense.	<i>Do the answers make sense?</i>
Answer Amanda is 22 years old, and her father is 44 years old.	

[From Lesson 1, Topic 1, Topic Text]

The last sentence in the problem asks for the ages of Amanda and her dad. Show students that this line alone is all they need to know that their final answer has to be "Amanda is ___ years old, and her father is ___ years old." Now they have a plan of attack—write an expression that describes each blank, and then combine the expressions into an equation to solve. This technique of writing the answer first helps students over the hump of knowing where to start with a word problem.

Students should be encouraged to clear decimals or fractions before solving any equation. This generally results in an equation that is easier to solve. You might want to demonstrate solving an equation both with and without clearing the fractions, and let the students see what a difference it makes.

Encourage students to always check their solution(s) by going back to the original equation or inequality. Sometimes they will want to substitute their solution into a more "friendly" equation that was derived from the original one. Since this can be misleading, especially if mistakes

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were made along the way, insist that they always plug their answers back into the original equation.

Difficult Solutions

In beginning math classes, problems are generally written to produce pleasant solutions—single, integer answers. As a result, some developmental math and algebra students are going to assume they did something wrong when their answers include variables, fractions, or decimals. Have them practice calculating and then checking complex solutions until they don't seem unusual.

Equations that have an infinite number of solutions or no solution will also be confusing. After you define and illustrate what these circumstances actually represent, you'll also need to go through the procedures for checking such answers for accuracy. For example, be sure to explain that if they get a result like $3 = 3$, it does not mean that $x = 3$. Nor does a result of $6 = 0$ mean that $x = 6$ or $x = 0$. In order to interpret their answers in this unit correctly, students will need to feel comfortable knowing that any equation can have one solution, no solution, or an infinite number of solutions.

Inequalities as answers also mean students need to consider a range of solutions instead of just one. Have them try at least two values when checking their work, the endpoint and a number included in the range. This procedure is shown in the following example:

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Example	
Problem	Solve for p. $-6p + 14 < -58$
$ \begin{array}{r} -6p + 14 < -58 \\ \underline{-14 \quad -14} \\ -6p > -72 \\ \underline{-6 \quad -6} \\ p > 12 \end{array} $	<p>Begin to isolate the variable by subtracting 14 from both sides of the inequality.</p> <p>Divide both sides of the inequality by -6 to express the variable with a coefficient of 1.</p> <p>Dividing by a negative number results in reversing the inequality sign.</p>
Check	<p>Check the solution.</p> <p>First, check the end point 12 in the related equation.</p>
$ \begin{array}{l} -6p + 14 = -58 \\ \text{yes } -6(12) + 14 = -58? \\ -72 + 14 = -58 \\ -58 = -58 \text{ Yes!} \end{array} $	<p>Then, try another value to check the inequality. Try 100.</p>
$ \begin{array}{l} -6p + 14 < -58 \\ -6(100) + 14 < -58 \\ -600 + 14 < -58 \\ -586 < -58 \text{ Yes!} \end{array} $	
Answer	$p > 12$

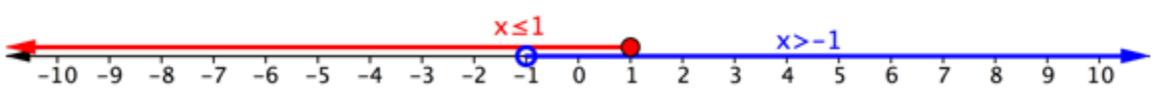
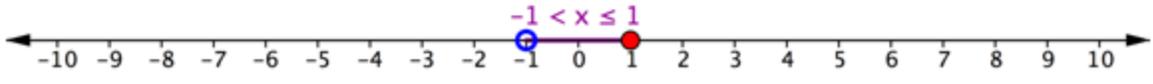
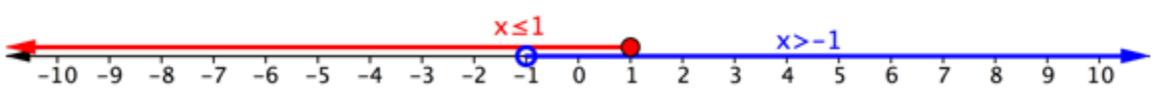
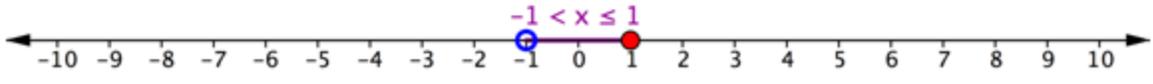
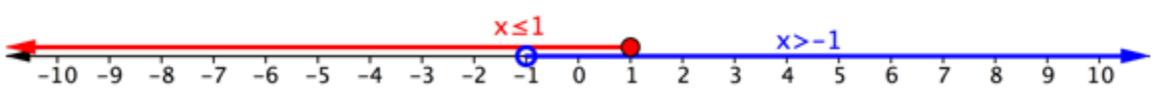
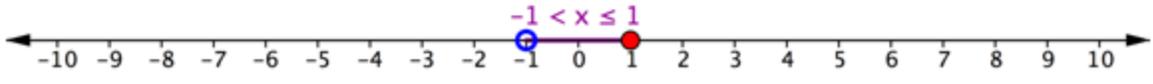
[From Lesson 2, Topic 2, Topic Text]

The And/Or Problem

Compound inequalities and absolute value problems can be bothersome topics for developmental math students, because they don't tend to differentiate between "and" and "or". You must lay out the mathematical difference clearly—"and" is the intersection of two sets (members in common) and "or" is the union of sets (all members of the sets). For example, have them think of an even number AND a counting number less than 10 (choices are 2, 4, 6, 8), then an even number OR a counting number less than 10 (choices are 1, 2, 3, 4, 5, 6, 7, 8, 9). Once students have those ideas firmly in mind, show them how they influence problem-solving.

In the case of inequalities, graphic illustrations will be very helpful driving the point home. Consider this example:

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Example																									
Problem	<p style="margin: 0;">Solve for x.</p> <p style="margin: 0;">$5x - 2 \leq 3$ and $4x + 7 > 3$</p>																								
	<table style="width: 100%; border: none;"> <tr> <td style="width: 45%; border: none;"> $\begin{array}{r} 5x - 2 \leq 3 \\ + 2 \quad + 2 \\ \hline 5x \leq 5 \\ \hline x \leq 1 \end{array}$ </td> <td style="width: 10%; border: none; text-align: center; vertical-align: middle;">AND</td> <td style="width: 45%; border: none;"> $\begin{array}{r} 4x + 7 > 3 \\ -7 \quad -7 \\ \hline 4x > -4 \\ \hline x > -1 \end{array}$ </td> </tr> <tr> <td colspan="3" style="border: none; padding: 5px;"> <p style="margin: 0; text-align: right;">Solve each inequality separately.</p> </td> </tr> <tr> <td colspan="3" style="border: none; padding: 5px;"> <p style="margin: 0; text-align: center;">$x > -1$ and $x \leq 1$</p> <p style="margin: 0; text-align: right;">Find the overlap between the solutions.</p> </td> </tr> <tr> <td colspan="3" style="border: none; padding: 5px;"> <p style="margin: 0;">The two inequalities can be represented graphically as:</p> </td> </tr> <tr> <td colspan="3" style="border: none; padding: 5px;">  </td> </tr> <tr> <td colspan="3" style="border: none; padding: 5px;"> <p style="margin: 0;">And the solution can be represented as:</p> </td> </tr> <tr> <td colspan="3" style="border: none; padding: 5px;">  </td> </tr> <tr> <td style="padding: 5px;">Answer</td> <td colspan="2" style="padding: 5px; text-align: center;">$x > -1$ and $x \leq 1$</td> </tr> </table>	$\begin{array}{r} 5x - 2 \leq 3 \\ + 2 \quad + 2 \\ \hline 5x \leq 5 \\ \hline x \leq 1 \end{array}$	AND	$\begin{array}{r} 4x + 7 > 3 \\ -7 \quad -7 \\ \hline 4x > -4 \\ \hline x > -1 \end{array}$	<p style="margin: 0; text-align: right;">Solve each inequality separately.</p>			<p style="margin: 0; text-align: center;">$x > -1$ and $x \leq 1$</p> <p style="margin: 0; text-align: right;">Find the overlap between the solutions.</p>			<p style="margin: 0;">The two inequalities can be represented graphically as:</p>						<p style="margin: 0;">And the solution can be represented as:</p>						Answer	$x > -1$ and $x \leq 1$	
$\begin{array}{r} 5x - 2 \leq 3 \\ + 2 \quad + 2 \\ \hline 5x \leq 5 \\ \hline x \leq 1 \end{array}$	AND	$\begin{array}{r} 4x + 7 > 3 \\ -7 \quad -7 \\ \hline 4x > -4 \\ \hline x > -1 \end{array}$																							
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Answer	$x > -1$ and $x \leq 1$																								

[From Lesson 3, Topic 1, Topic Text]

With this type of problem, students easily solve the two individual inequalities, but then they stop there, forgetting they are looking for the overlap rather than the combination of solutions. By having them sketch each individual inequality as well as the solution, they'll see how important the "and" really is.

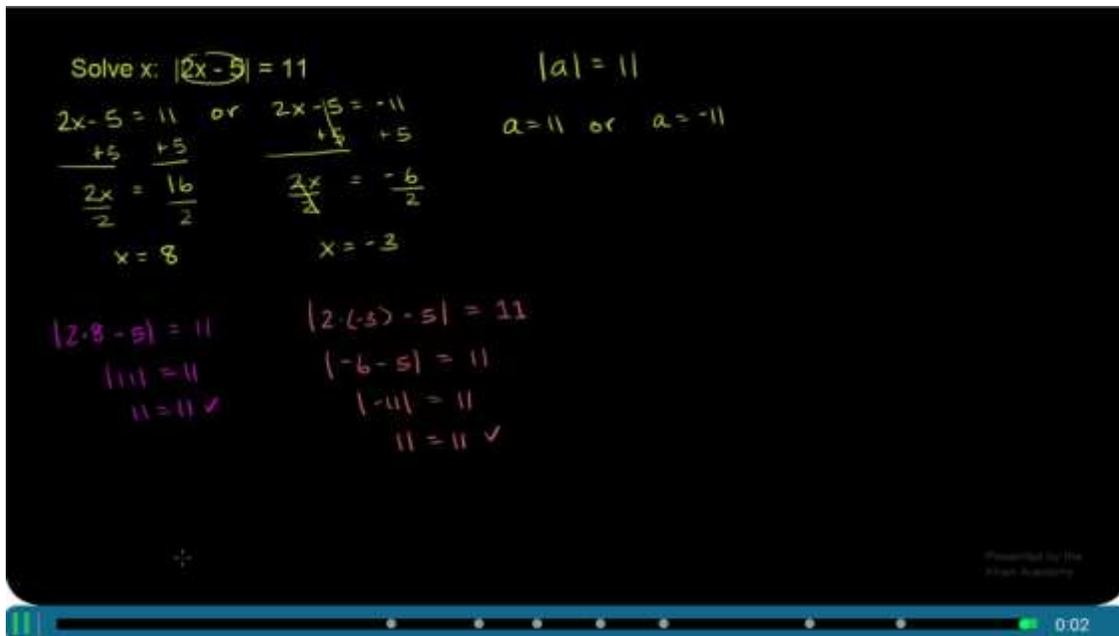
Students also struggle with "and" when solving an inequality of the form $a < x < c$. Because this concept is taught with compound conjunctions, they tend to break this apart into two inequalities: $a < x$ as well as $x < c$. Then they either put "or" instead of "and" between the two inequalities or leave out a conjunction altogether. We suggest avoiding the problem altogether by teaching your students how to solve these inequalities as a whole:

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Example	
Problem	<p style="text-align: center;">Solve for x. $3 < 2x + 3 \leq 7$</p>
	$\begin{array}{r} 3 < 2x + 3 \leq 7 \\ -3 \quad -3 \quad -3 \\ \hline 0 < 2x \leq 4 \\ 2 \quad 2 \quad 2 \\ \hline 0 < x \leq 2 \end{array}$ <p style="font-size: small;">Isolate the variable by subtracting 3 from all 3 parts of the inequality, and then dividing each part by 2.</p>
Answer	$0 < x \leq 2$

[From Lesson 3, Topic 1, Topic Text]

Just like with compound inequalities, students confuse “and” with “or” when solving absolute value problems. Emphasize the importance of actually writing down “or” in between the two possibilities, as seen here:



[From Lesson 3, Topic 2, Worked Example 1]

Keep in Mind

Material in this unit has been geared to both the beginning and intermediate developmental math student. More difficult examples and problems are included for intermediate students, but

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these could be used to challenge beginners as well, if appropriate. The topics of compound inequalities and absolute value equations and inequalities are included but are not intended for the beginning algebra student.

When graphing an inequality on a number line in beginning algebra, open and closed circles are used to indicate endpoints. In intermediate algebra, sometimes brackets and parentheses are used to indicate endpoints, especially when graphing compound inequalities.

Additional Resources

In all mathematics, the best way to really learn new skills and ideas is repetition. Problem solving is woven into every aspect of this course—each topic includes warm-up, practice, and review problems for students to solve on their own. The presentations, worked examples, and topic texts demonstrate how to tackle even more problems. But practice makes perfect, and some students will benefit from additional work.

For more practice simplifying expressions, go to <http://www.mathsnet.net/algebra/a31.html>.

A balance scale applet at http://nlvm.usu.edu/en/nav/frames_asid_201_g_4_t_2.html?open=instructions illustrates how an equation must always be in balance.

A good site for algebra review is www.mathsnet.net – see additional problems by clicking on “more on this topic”.

Practice translating a number word problem into an equation and then solving it at <http://www.mathsnet.net/algebra/c11.html>.

Solve formulas for a specific letter at <http://www.mathsnet.net/algebra/c21.html> (the site will ask you to rearrange a formula to make x the subject – this means to solve the formula for x .)

Practice solving linear inequalities and compound inequalities at <http://www.mathsnet.net/algebra/c31.html>.

Summary

This unit provides the foundation for understanding algebra. It teaches students the mechanics of solving equations and inequalities, and explains the principles behind these operations and procedures. After completing this portion of the course, beginning students will be able to simplify and solve linear equations and inequalities. Intermediate students will also know how to work with compound inequalities and absolute value problems.

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Unit 10 – Tutor Simulation

Unit 10: Solving Equations and Inequalities

Instructor Overview

Tutor Simulation: Building A Dog Kennel

Purpose

This simulation allows students to demonstrate their ability to write and solve equations and inequalities. Students will be asked to apply what they have learned to solve a problem involving:

- Writing equations
- Solving equations
- Writing inequalities
- Solving inequalities
- Representing inequalities on a number line
- Using formulas

Problem

Students are presented with the following problem:

A property owner wants to build a dog kennel on her property so her dog has plenty of room to run. Before she builds it, she has a number of size, material, and price options to consider. You will use your knowledge of equations and inequalities to help her figure out what sizes and materials meet her needs and fit her budget.

Recommendations

Tutor simulations are designed to give students a chance to assess their understanding of unit material in a personal, risk-free situation. Before directing students to the simulation,

- Make sure they have completed all other unit material.
- Explain the mechanics of tutor simulations.
 - Students will be given a problem and then guided through its solution by a video tutor;
 - After each answer is chosen, students should wait for tutor feedback before continuing;
 - After the simulation is completed, students will be given an assessment of their efforts. If areas of concern are found, the students should review unit materials or seek help from their instructor.
- Emphasize that this is an exploration, not an exam.

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Unit 10 – Puzzle

Unit 10: Solving Equations and Inequalities

Instructor Overview Puzzle: What's More?

Objectives

What's More is a puzzle that asks players to assess the weights of fruits on balance scales. Students must be able to recognize and write inequalities, and then apply the properties of inequality in order to choose the heaviest fruit.



Figure 1. *What's More?* offers an unbalanced diet of fruit so that the learner can practice working with inequalities.

Description

In each level, players see multiple scales holding various fruits. They have to identify the heaviest in each pair, and then combine all the individual inequalities to find out which fruit is the heaviest of them all. After a player makes a choice, the inequality representing the relationship of the fruits is shown to reinforce the analysis. Players score points for solving each puzzle and the points accumulate as the game proceeds.

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There are three levels of difficulty. In level one, there are two scales and three fruits. In level two, four fruits are arranged on three scales. In level three, four fruits are grouped in combinations on three scales. Each level includes ten puzzles that are generated in real time, so the game can be played over and over.

What's More? is suitable for both individual and group play. The game can also be run in a classroom setting to provoke interest in the topic and to allow students to discuss their reasoning in comparing inequalities.

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Unit 10 – Project

Unit 10: Solving Equations and Inequalities

Instructor Overview Project: Silkscreen Start-Up

Student Instructions

Introduction

There are many things to consider when opening a business. Here, you will use your ability to set up and solve equations and inequalities to determine whether it is profitable to start a new business.

Task

In this project you will play the part of a consultant to a prospective business owner. Working together with your group, you will gather data and make calculations to determine the costs associated with opening a silk-screen printing business. You will make a final recommendation to the prospective owner.

Instructions

Solve each problem in order and save your work along the way, as you will create a professional report at the conclusion of the project. If required, round to the hundredths place (two decimal places) for quantities of money and to the nearest whole number otherwise.

- First problem:
 - The costs of operating a business can be divided into overhead and operating costs. The overhead costs involve items that are simply needed for the business to exist, regardless of whether it produces anything or not. The operating costs result from actually running equipment or using materials, and these vary depending on how much the company produces. In the chart below are some possible overhead costs. From the web, local newspapers, or your own personal experience, find the reasonable costs for these items in your own town and record them in the chart below.

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Overhead Cost Item	Cost in Dollars (per month)
Rent	
Utilities	
Water	
Gas	
Electric	
Phone	
Your Monthly Salary & Benefits	
Property Insurance	
Advertising Costs	
Other Costs?	
TOTAL=	

- Second Problem:
 - One of the main costs of your business will be the purchase of equipment. For each job, a silk screen must be set up regardless of how many prints are made, and once the silk screen is set up, each print costs a certain amount because of the ink and wear-and-tear on the machine.

There are two different machines to choose from:

Machine	Cost of Setting Up Screen	Cost of Ink Etc. per Print
A	\$20.00	\$1.30
B	\$75.00	\$0.80

- Now, for each machine, write an expression for the cost of producing q prints. (q stands for quantity.)

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Machine	Cost of Producing q Prints
A	
B	

- Depending on how many prints we wish to make, one of the machines will be less expensive to use. Write and solve an inequality to determine the values of q for which Machine A will be the less expensive option.
- Third Problem:
 - In order to decide which machine to buy, you need to know the size of an average job. Suppose that market research indicates that in your location you could expect an average job size of $q=45$ prints. Determine which machine to buy and your cost per job.
 - Now, write an expression for the cost of processing j jobs.
 - Now, write an expression for the: overhead costs + cost of j jobs. This will involve your overhead costs from Problem 1.
 - Suppose market research shows that the maximum price people are willing to pay for a print is \$7.00, or $45 \times 7 = \$315.00$ per job. Write an expression for the amount of income you will get from j jobs.
 - Finally, write and solve an equation to determine how many jobs will be needed to break even for the month. Remember that breaking even means that your cost of processing j jobs equals your income from them. (*Hint*: remember that you cannot realistically process part of a job, so the number of jobs j will have to be a whole number.)
- Fourth Problem (Optional)
 - To add a dose of realism to your perspective, find a local owner of a small business and conduct an interview with them:
 - Ask them how they determine their overhead costs and their profit.
 - How did they determine whether it was reasonable to open their business?
 - How much does the amount of their income change from month to month?
 - Use this information when you write your recommendation so that it sounds realistic and informs the prospective owner of other things that need to be considered when opening a business.

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Collaboration

Get together with another group to compare your answers to each of the four problems. Discuss how you might combine your answers to make a more complete and convincing analysis of the situation.

Conclusions

Finally, present your solution as a “Feasibility Study” to the prospective business owner. Be sure to clearly explain your reasoning at each stage and conclude with a recommendation to either pursue the start-up or not depending on how many jobs the market will support.

Instructor Notes

Assignment Procedures

Problem 1

Of course the answers the students collect will be varied. For the purposes of illustration, we offer the following values and will use them throughout to compute our answers. Students may or may not think of other fixed costs.

Overhead Cost Item	Cost in Dollars (per month)
Rent	\$1,100.00
Utilities	-
Water	\$30.00
Gas	\$85.00
Electric	\$135.00
Phone	\$70.00
Your Monthly Salary & Benefits	\$4,200.00
Property Insurance	\$145.00
Advertising Costs	\$200.00
Other Costs?	-

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TOTAL=	\$5,965.00
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Problem 2

The answers are shown in the table below.

Machine	Cost of Producing q Prints
A	$1.30q+20$
B	$0.80q+75$

Cost using A must be less than or equal to Cost using B, so the inequality is $1.30q+20 < 0.80q+75$, and so $q < 110$.

Problem 3

Since $45 < 110$, we would lease Machine A, and the cost per job would be $1.30 \times 45 + 75 = \$78.50$.

Since each job would cost \$78.50, the cost would be $78.50j$.

Answers here will vary since each student will have slightly different overhead costs. Our expression is $5965+78.50j$.

$315j$.

[Answers will vary due to that fact that each student will have slightly different overhead costs. With our data, we need to solve $5965+78.50j=315j$ to obtain 25.2, but since we cannot process partial jobs, we need 26 jobs per month.

Recommendations

- Have students work in teams to encourage brainstorming and cooperative learning.
- Assign a specific timeline for completion of the project that includes milestone dates.
- Provide students feedback as they complete each milestone.
- Ensure that each member of student groups has a specific job.

Technology Integration

This project provides abundant opportunities for technology integration, and gives students the chance to research and collaborate using online technology. The students' instructions list several websites that provide information on numbering systems, game design, and graphics.

The following are other examples of free Internet resources that can be used to support this project:

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<http://www.moodle.org>

An Open Source Course Management System (CMS), also known as a Learning Management System (LMS) or a Virtual Learning Environment (VLE). Moodle has become very popular among educators around the world as a tool for creating online dynamic websites for their students.

<http://www.wikispaces.com/site/for/teachers> or <http://pbworks.com/content/edu+overview>

Allows you to create a secure online Wiki workspace in about 60 seconds. Encourage classroom participation with interactive Wiki pages that students can view and edit from any computer. Share class resources and completed student work with parents.

<http://www.docs.google.com>

Allows students to collaborate in real-time from any computer. Google Docs provides free access and storage for word processing, spreadsheets, presentations, and surveys. This is ideal for group projects.

<http://why.openoffice.org/>

The leading open-source office software suite for word processing, spreadsheets, presentations, graphics, databases and more. It can read and write files from other common office software packages like Microsoft Word or Excel and MacWorks. It can be downloaded and used completely free of charge for any purpose.

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Rubric

Score	Content	Presentation/Communication
4	<ul style="list-style-type: none"> The solution shows a deep understanding of the problem including the ability to identify the appropriate mathematical concepts and the information necessary for its solution. The solution completely addresses all mathematical components presented in the task. The solution puts to use the underlying mathematical concepts upon which the task is designed and applies procedures accurately to correctly solve the problem and verify the results. Mathematically relevant observations and/or connections are made. 	<ul style="list-style-type: none"> There is a clear, effective explanation detailing how the problem is solved. All of the steps are included so that the reader does not need to infer how and why decisions were made. Mathematical representation is actively used as a means of communicating ideas related to the solution of the problem. There is precise and appropriate use of mathematical terminology and notation. Your project is professional looking with graphics and effective use of color.
3	<ul style="list-style-type: none"> The solution shows that the student has a broad understanding of the problem and the major concepts necessary for its solution. The solution addresses all of the mathematical components presented in the task. The student uses a strategy that includes mathematical procedures and some mathematical reasoning that leads to a solution of the problem. Most parts of the project are correct with only minor mathematical errors. 	<ul style="list-style-type: none"> There is a clear explanation. There is appropriate use of accurate mathematical representation. There is effective use of mathematical terminology and notation. Your project is neat with graphics and effective use of color.
2	<ul style="list-style-type: none"> The solution is not complete indicating that parts of the problem are not understood. The solution addresses some, but not all of the mathematical components presented in the task. The student uses a strategy that is partially useful, and demonstrates some evidence of mathematical reasoning. Some parts of the project may be correct, but major errors are noted and the student could not completely carry out mathematical procedures. 	<ul style="list-style-type: none"> Your project is hard to follow because the material is presented in a manner that jumps around between unconnected topics. There is some use of appropriate mathematical representation. There is some use of mathematical terminology and notation appropriate to the problem. Your project contains low quality graphics and colors that do not add interest to the project.
1	<ul style="list-style-type: none"> There is no solution, or the solution has no relationship to the task. No evidence of a strategy, procedure, or mathematical reasoning and/or uses a strategy that does not help solve the problem. 	<ul style="list-style-type: none"> There is no explanation of the solution, the explanation cannot be understood or it is unrelated to the problem. There is no use or inappropriate use of mathematical representations (e.g.

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	<ul style="list-style-type: none">• The solution addresses none of the mathematical components presented in the task.• There were so many errors in mathematical procedures that the problem could not be solved.	<p>figures, diagrams, graphs, tables, etc.).</p> <ul style="list-style-type: none">• There is no use, or mostly inappropriate use, of mathematical terminology and notation.• Your project is missing graphics and uses little to no color.
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Unit 10 – Correlation to Common Core Standards

Unit 10: Solving Equations and Inequalities

Common Core Standards

Unit 10, Lesson 1, Topic 1: Solving One-Step Equations Using Properties of Equality

Grade: 8 - Adopted 2010

STRAND / DOMAIN	CC.MP.	Mathematical Practices
CATEGORY / CLUSTER	MP.1.	Make sense of problems and persevere in solving them.
STRAND / DOMAIN	CC.8.EE.	Expressions and Equations
CATEGORY / CLUSTER		Analyze and solve linear equations and pairs of simultaneous linear equations.
STANDARD	8.EE.7.	Solve linear equations in one variable.
EXPECTATION	8.EE.7(a)	Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). (SBAC Summative Assessment Target: 1.06, 2.02, 3.03, 4.01)
EXPECTATION	8.EE.7(b)	Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. (SBAC Summative Assessment Target: 1.06, 2.02, 3.03, 4.01)

Grade: 9-12 - Adopted 2010

STRAND / DOMAIN	CC.MP.	Mathematical Practices
CATEGORY / CLUSTER	MP.1.	Make sense of problems and persevere in solving them.
STRAND / DOMAIN	CC.A.	Algebra
CATEGORY / CLUSTER	A-REI.	Reasoning with Equations and Inequalities
STANDARD		Understand solving equations as a process of reasoning and explain the reasoning.
EXPECTATION	A-REI.1.	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
STRAND / DOMAIN	CC.A.	Algebra
CATEGORY / CLUSTER	A-REI.	Reasoning with Equations and Inequalities
STANDARD		Solve equations and inequalities in one variable.

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EXPECTATION	A-REI.3.	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
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Unit 10, Lesson 1, Topic 2: Solving Multi-Step Equations

Grade: 8 - Adopted 2010

STRAND / DOMAIN	CC.MP.	Mathematical Practices
CATEGORY / CLUSTER	MP.1.	Make sense of problems and persevere in solving them.
STRAND / DOMAIN	CC.8.EE.	Expressions and Equations
CATEGORY / CLUSTER		Analyze and solve linear equations and pairs of simultaneous linear equations.
STANDARD	8.EE.7.	Solve linear equations in one variable.
EXPECTATION	8.EE.7(a)	Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). (SBAC Summative Assessment Target: 1.06, 2.02, 3.03, 4.01)
EXPECTATION	8.EE.7(b)	Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. (SBAC Summative Assessment Target: 1.06, 2.02, 3.03, 4.01)

Grade: 9-12 - Adopted 2010

STRAND / DOMAIN	CC.MP.	Mathematical Practices
CATEGORY / CLUSTER	MP.1.	Make sense of problems and persevere in solving them.
STRAND / DOMAIN	CC.A.	Algebra
CATEGORY / CLUSTER	A-REI.	Reasoning with Equations and Inequalities
STANDARD		Understand solving equations as a process of reasoning and explain the reasoning.
EXPECTATION	A-REI.1.	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
STRAND / DOMAIN	CC.A.	Algebra
CATEGORY / CLUSTER	A-REI.	Reasoning with Equations and Inequalities
STANDARD		Solve equations and inequalities in one variable.
EXPECTATION	A-REI.3.	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Unit 10, Lesson 1, Topic 3: Special Cases and Applications

Grade: 8 - Adopted 2010

STRAND / DOMAIN	CC.MP.	Mathematical Practices
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CATEGORY / CLUSTER	MP.1.	Make sense of problems and persevere in solving them.
STRAND / DOMAIN	CC.8.EE.	Expressions and Equations
CATEGORY / CLUSTER		Analyze and solve linear equations and pairs of simultaneous linear equations.
STANDARD	8.EE.7.	Solve linear equations in one variable.
EXPECTATION	8.EE.7(a)	Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). (SBAC Summative Assessment Target: 1.06, 2.02, 3.03, 4.01)
EXPECTATION	8.EE.7(b)	Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. (SBAC Summative Assessment Target: 1.06, 2.02, 3.03, 4.01)

Grade: **9-12** - Adopted **2010**

STRAND / DOMAIN	CC.MP.	Mathematical Practices
CATEGORY / CLUSTER	MP.1.	Make sense of problems and persevere in solving them.
STRAND / DOMAIN	CC.A.	Algebra
CATEGORY / CLUSTER	A-REI.	Reasoning with Equations and Inequalities
STANDARD		Understand solving equations as a process of reasoning and explain the reasoning.
EXPECTATION	A-REI.1.	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
STRAND / DOMAIN	CC.A.	Algebra
CATEGORY / CLUSTER	A-REI.	Reasoning with Equations and Inequalities
STANDARD		Solve equations and inequalities in one variable.
EXPECTATION	A-REI.3.	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Unit 10, Lesson 1, Topic 4: Formulas

Grade: **8** - Adopted **2010**

STRAND / DOMAIN	CC.MP.	Mathematical Practices
CATEGORY / CLUSTER	MP.1.	Make sense of problems and persevere in solving them.

Grade: **9-12** - Adopted **2010**

STRAND / DOMAIN	CC.MP.	Mathematical Practices
CATEGORY / CLUSTER	MP.1.	Make sense of problems and persevere in solving them.
STRAND / DOMAIN	CC.A.	Algebra

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CATEGORY / CLUSTER	A-CED.	Creating Equations
STANDARD		Create equations that describe numbers or relationships.
EXPECTATION	A-CED.4.	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law $V = IR$ to highlight resistance R .

Unit 10, Lesson 2, Topic 1: Solving One-Step Inequalities

Grade: 8 - Adopted 2010

STRAND / DOMAIN	CC.MP.	Mathematical Practices
CATEGORY / CLUSTER	MP.1.	Make sense of problems and persevere in solving them.

Grade: 9-12 - Adopted 2010

STRAND / DOMAIN	CC.MP.	Mathematical Practices
CATEGORY / CLUSTER	MP.1.	Make sense of problems and persevere in solving them.
STRAND / DOMAIN	CC.A.	Algebra
CATEGORY / CLUSTER	A-REI.	Reasoning with Equations and Inequalities
STANDARD		Solve equations and inequalities in one variable.
EXPECTATION	A-REI.3.	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Unit 10, Lesson 2, Topic 2: Multi-Step Inequalities

Grade: 8 - Adopted 2010

STRAND / DOMAIN	CC.MP.	Mathematical Practices
CATEGORY / CLUSTER	MP.1.	Make sense of problems and persevere in solving them.

Grade: 9-12 - Adopted 2010

STRAND / DOMAIN	CC.MP.	Mathematical Practices
CATEGORY / CLUSTER	MP.1.	Make sense of problems and persevere in solving them.
STRAND / DOMAIN	CC.A.	Algebra
CATEGORY / CLUSTER	A-REI.	Reasoning with Equations and Inequalities
STANDARD		Solve equations and inequalities in one variable.
EXPECTATION	A-REI.3.	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Unit 10, Lesson 3, Topic 1: Compound Inequalities

Grade: 8 - Adopted 2010

STRAND / DOMAIN	CC.MP.	Mathematical Practices
CATEGORY / CLUSTER	MP.1.	Make sense of problems and persevere in solving them.

Grade: 9-12 - Adopted 2010

STRAND / DOMAIN	CC.MP.	Mathematical Practices
CATEGORY / CLUSTER	MP.1.	Make sense of problems and persevere in solving them.

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STRAND / DOMAIN	CC.A.	Algebra
CATEGORY / CLUSTER	A-REI.	Reasoning with Equations and Inequalities
STANDARD		Solve equations and inequalities in one variable.
EXPECTATION	A-REI.3.	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Unit 10, Lesson 3, Topic 2: Equations and Inequalities and Absolute Value

Grade: **8** - Adopted **2010**

STRAND / DOMAIN	CC.MP.	Mathematical Practices
CATEGORY / CLUSTER	MP.1.	Make sense of problems and persevere in solving them.

Grade: **9-12** - Adopted **2010**

STRAND / DOMAIN	CC.MP.	Mathematical Practices
CATEGORY / CLUSTER	MP.1.	Make sense of problems and persevere in solving them.
STRAND / DOMAIN	CC.A.	Algebra
CATEGORY / CLUSTER	A-REI.	Reasoning with Equations and Inequalities
STANDARD		Solve equations and inequalities in one variable.
EXPECTATION	A-REI.3.	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.