Unit 12: Introduction to Factoring

Unit 12: Factoring

Lesson 1: Introduction to Factoring

Topic 1: Greatest Common Factor

Learning Objectives

- Find the greatest common factor (GCF) of monomials.
- Factor polynomials by factoring out the greatest common factor (GCF).
- Factor expressions with four terms by grouping.

Lesson 2: Factoring Polynomials

Topic 1: Factoring Trinomials

Learning Objectives

- Factor trinomials with a leading coefficient of 1.
- Factor trinomials with a common factor.
- Factor trinomials with a leading coefficient other than 1.

Topic 2: Factoring: Special Cases

Learning Objectives

- Factor trinomials that are perfect squares.
- Factor binomials in the form of the difference of squares.

Topic 3: Special Cases: Cubes

Learning Objectives

- Factor the sum of cubes.
- Factor the difference of cubes.

Lesson 3: Solving Quadratic Equations

Topic 1: Solve Quadratic Equations by Factoring

Learning Objectives

- Solve equations in factored form by using the Principle of Zero Products.
- Solve quadratic equations by factoring and then using the Principle of Zero Products.
- Solve application problems involving quadratic equations.

Unit 12: Factoring

Instructor Notes

The Mathematics of Factoring

This unit builds upon students' knowledge of polynomials learned in the previous unit. They will learn how to use the distributive property and greatest common factors to find the factored form of binomials and how to factor trinomials by grouping. Students will also learn how to recognize and quickly factor special products (perfect square trinomials, difference of squares, and the sum and difference of two squares). Finally, they'll get experience combining these techniques and using them to solve quadratic equations.

Teaching Tips: Challenges and Approaches

This unit on factoring is probably one of the most difficult—students will spend a lot of time carrying out multi-step, complex procedures for what will often seem to be obscure purposes. At this stage in algebra, factoring polynomials may feel like busy work rather than a means to a useful end. It doesn't help that students may remember having trouble with factoring from when they studied algebra in high school.

Encourage students to think of factoring as the reverse of multiplying polynomials that was learned previously. Then, a problem multiplying polynomials was given and students were asked to calculate the answer. In this unit, the answer is given and the students need to come up with the question. Sound familiar? In a way, factoring is like playing the popular game show *Jeopardy*.

Greatest Common Factor

Finding the greatest common factor of whole numbers should be reviewed before finding the GCF of polynomials. Then it is a logical step to demonstrate how to factor expressions by using the distributive property in reverse to pull out the greatest common monomial from each term in a polynomial:

Example		
Problem	**Factor $81c^3d + 45c^2d^2$.**	
	$3 \cdot 3 \cdot 9 \cdot c \cdot c \cdot c \cdot d$	Factor $81c^3d$.
	$3 \cdot 3 \cdot 5 \cdot c \cdot c \cdot d \cdot d$	Factor $45c^2d^2$.
	$3 \cdot 3 \cdot c \cdot c \cdot d = 9c^2d$	Find the GCF.
	$81c^3d = 9c^2d(9c)$ $45c^2d^2 = 9c^2d(5d)$	Rewrite each term as the product of the GCF and the remaining terms.
	$9c^2d(9c) + 9c^2d(5d)$	Rewrite the polynomial expression using the factored terms in place of the original terms.
	$9c^2d(9c + 5d)$	Factor out $9c^2d$.
Answer	$9c^2d(9c + 5d)$	

[From Lesson 1, Topic 1, Topic Text]

Remind your students to pay particular attention to signs as it is easy to make a mistake with them, and also to check their final answers by multiplying.

Grouping

After your students are comfortable pulling the GCF out of a polynomial, it is time to teach them a new method of factoring–factoring by grouping. Begin by introducing the technique on 4-term polynomials. It's fairly easy for students to understand how to break these polynomials into groups of two and then factor each pair.

Trinomials are trickier. Indeed, many textbooks do not use grouping for factoring trinomials, and instead use essentially a guess and check method. While factoring by grouping may initially be a more complex procedure, it has many significant advantages in the long term and is used in this course. The hardest part is figuring out how to rewrite the middle term of a trinomial as an equivalent binomial. Students will need to see this demonstrated repeatedly, as well as get a lot of practice working on their own. Even after they grasp the basic idea, they'll often have trouble deciding which signs to use. It will be helpful to supply them with a set of tips like the one below:

Tips for Finding Values that Work

When factoring a trinomial in the form $x^2 + bx + c$, consider the following tips.

Look at the *c* term first.

- If the *c* term is a positive number, then the factors of *c* will both be positive or both be negative. In other words, *r* and *s* will have the same sign.
- If the *c* term is a negative number, then one factor of *c* will be positive, and one factor of *c* will be negative. Either *r* or *s* will be negative, but not both.

Look at the *b* term second.

- If the *c* term is positive and the *b* term is positive, then both *r* and *s* are positive.
- If the *c* term is positive and the *b* term is negative, then both *r* and *s* are negative.
- If the *c* term is negative and the *b* term is positive, then the factor that is positive will have the greater absolute value. That is, if $|r| > |s|$, then *r* is positive and *s* is negative.
- If the *c* term is negative and the *b* term is negative, then the factor that is negative will have the greater absolute value. That is, if $|r| > |s|$, then *r* is negative and *s* is positive.

[From Lesson 2, Topic 1, Topic Text]

Factoring by grouping has the great advantage of working for all trinomials. It also provides a method to determine when a polynomial cannot be factored. (This is not obvious when students are using the guess and check method.)

Sometimes students don't remember to look for the greatest common factor of all the terms of a polynomial before trying to factor by grouping. This isn't wrong, but the larger numbers can make the work more difficult. Plus the student has to remember to look for a greatest common factor at the end anyway. In order to illustrate this, have students factor $9x^2 + 15x - 36$ without pulling out the greatest common factor of 3 -- they will notice that the numbers are cumbersome. After this, have them try again, this time factoring out the 3 as the first step. They will see the benefits.

Once the grouping method is mastered, let your students use it to factor perfect square trinomials. Hopefully they'll soon see a pattern, though you will probably have to nudge them along. Eventually, they should learn to recognize if a trinomial is a perfect square, and be able to factor it without grouping.

After the rule for factoring a perfect square trinomial has been developed, set them to finding one for factoring the difference of two squares. This rule is usually very easy for students to figure out. Then have them try to factor the sum of two squares, such as $x^2 + 4$. Make sure they understand that this cannot be done.

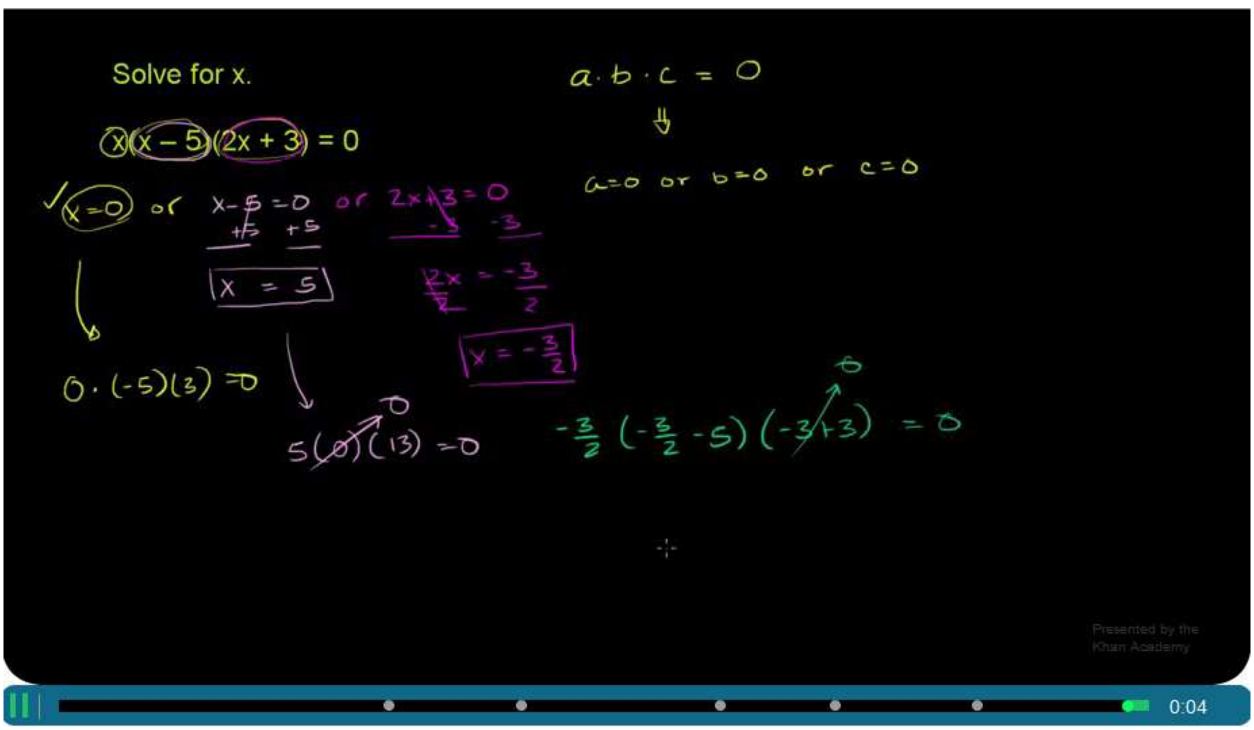
Intermediate algebra students will also need to know how to factor the sum and difference of two cubes. They are sure to have trouble remembering the formulas. Try pointing out that the formulas are really the same except for signs:

- A binomial in the form $a^3 + b^3$ can be factored as $(a + b)(a^2 – ab + b^2)$
- A binomial in the form $a^3 – b^3$ can be factored as $(a – b)(a^2 + ab + b^2)$

The sign in between the two cubes is the same sign as in the first factor in the formulas. The next sign is the opposite of the first sign and the last sign is always positive. Now "all" they have to remember are the variable parts of the formulas. Easy!

Factoring Quadratic Equations

The last topic in this unit is solving quadratic equations by factoring and applying the zero products rule. Begin by solving an example where the polynomial is already factored and set equal to zero, such as the following:



[From Lesson 3, Topic 1, Worked Example 1]

Now give your students a problem like "Solve $x^2 + x – 12 = 0$ for x." Ask them how they would attempt to solve for x. Someone will suggest factoring the left hand side by grouping and they will be on their way.

Then pose the problem $x^2 + x – 12 = 18$. Make sure your students know that in order for the principle of zero products to work, the trinomial must be set equal to 0. Sometimes students are so focused on new techniques, they forget basic principles for rewriting an equation and they may need to be prodded to add (or subtract) something to (or from) both sides so that one side equals zero.

Be careful -- once students get into the hang of applying the zero products rule to solve equations, they may start trying it on expressions as well. For instance, if a problem says to factor $x^2 + x - 12$, some will do so and then go ahead and calculate that $x = -4$ or 3. Remind your students to only do what a problem asks – factor when it says to factor and solve when it says to solve.

The Sense Test

Application problems have an extra requirement that solving given equations do not -- answers have to make sense based on their context. Consider the following scenario:

Example		
Problem	**A small toy rocket is launched from a 4-foot pedestal. The height (h, in feet) of the rocket t seconds after taking off is given by the formula $h = -2t^2 + 7t + 4$. How long will it take the rocket to hit the ground?**	
	$h = -2t^2 + 7t + 4$	The rocket will be on the ground when the height is 0. So, substitute 0 for h in the formula.
	$0 = -2t^2 + 7t + 4$	
	$0 = -2t^2 + 8t - t + 4$	Factor the trinomial by grouping.
	$0 = -2t(t-4) - 1(t-4)$ $0 = (-2t - 1)(t-4)$ $0 = -1(2t+1)(t-4)$	Factor.
	$2t + 1 = 0$ or $t - 4 = 0$	Use the Zero Product Property.
	$t = -\frac{1}{2}$ or $t = 4$	Solve each equation.
	$t = 4$	Interpret the answer. Since t represents time, it cannot be a negative number; only $t = 4$ makes sense in this context.
Answer	The rocket will hit the ground 4 seconds after being launched.	

[From Lesson 3, Topic 1, Topic Text]

Mathematically, it is true that t can be either 4 or $-\frac{1}{2}$. But logically, only one of these answers works -- since t represents the number of seconds after the rocket has taken off, it can't be a negative number. The rocket can't hit the ground before it was launched. Teach students that when they do application problems like this, they need to check not only the math but also the sense of their results.

Keep in Mind

Factoring trinomials and solving quadratic equations are difficult topics. As soon as you say "factoring," some students will recall hours of erasing before correct answers were found through trial and error. Reassure students that while the factoring by grouping method takes longer to use when working simple problems, it really will make solving complex problems quicker. Stress to your students that once something is factored, they should check their work by multiplying. This will help them catch any errors that were made.

Most of the material in this unit has been geared to both beginning and intermediate students. More difficult examples and problems are included for the intermediate algebra student, but these could be used to challenge the beginning algebra student. However, two topics, factoring the sum and difference of two cubes, are intended only for intermediate algebra students.

Additional Resources

In all mathematics, the best way to really learn new skills and ideas is repetition. Problem solving is woven into every aspect of this course—each topic includes warm-up, practice, and review problems for students to solve on their own. The presentations, worked examples, and topic texts demonstrate how to tackle even more problems. But practice makes perfect, and some students will benefit from additional work.

Practice finding the common factor of polynomials at http://www.mathsnet.net/algebra/a41.html (get additional problems by clicking on "more on this topic").

Factoring practice using the AC Method can be found at http://www.ltcconline.net/greenl/java/BasicAlgebra/AC/AC.html.

Solve quadratic equations using the principle of zero products at http://www.mathsnet.net/algebra/e34.html (get additional problems by clicking on "more on this topic").

Practice all types of factoring problems at http://www.hostsrv.com/webmab/app1/MSP/quickmath/02/pageGenerate?site=quickmath&s1=algebra&s2=factor&s3=basic.

Review factoring and solving quadratic equations at http://www.quia.com/rr/36611.html.

Summary

After completing this unit, students will be more comfortable with factoring any polynomial that is given to them. They'll be able to pull out the GCF and factor by grouping, and recognize special cases such as perfect square trinomials, difference of two squares, and the sum and difference of two cubes. They'll have had experience combining these techniques to solve quadratic equations, and will have gained an appreciation that factoring can be used to solve real-life problems.

Unit 12: Factoring

Instructor Overview
Tutor Simulation: Playing the Elimination Game

Purpose

This simulation allows students to demonstrate their ability to factor polynomials. Students will be asked to apply what they have learned to solve a problem involving:

- Factoring out the greatest common factor
- Factoring by grouping
- Factoring the sum or difference of perfect squares
- Factoring the sum or difference of cubes

Problem

Students are presented with the following problem:

> Many mathematicians have tricks they use to analyze an expression in order to determine if they can factor it quickly. In this simulation, we are going to focus on techniques for quickly factoring polynomials with two terms (binomials) or with four terms.

Recommendations

Tutor simulations are designed to give students a chance to assess their understanding of unit material in a personal, risk-free situation. Before directing students to the simulation,

- Make sure they have completed all other unit material.
- Explain the mechanics of tutor simulations.
    - Students will be given a problem and then guided through its solution by a video tutor;
    - After each answer is chosen, students should wait for tutor feedback before continuing;
    - After the simulation is completed, students will be given an assessment of their efforts. If areas of concern are found, the students should review unit materials or seek help from their instructor.
- Emphasize that this is an exploration, not an exam.

Unit 12: Factoring

Instructor Overview
Puzzle: Match Factors

Objectives

Match Factors is a puzzle that tests a player's ability to factor by grouping. It reinforces the technique of factoring a trinomial in the form $ax^2 + bx + c$ by finding two integers, r and s, whose sum is b and whose product is ac. Puzzle play, especially when done by eye rather than with pencil and paper, will help students learn to quickly identify the components of factors.

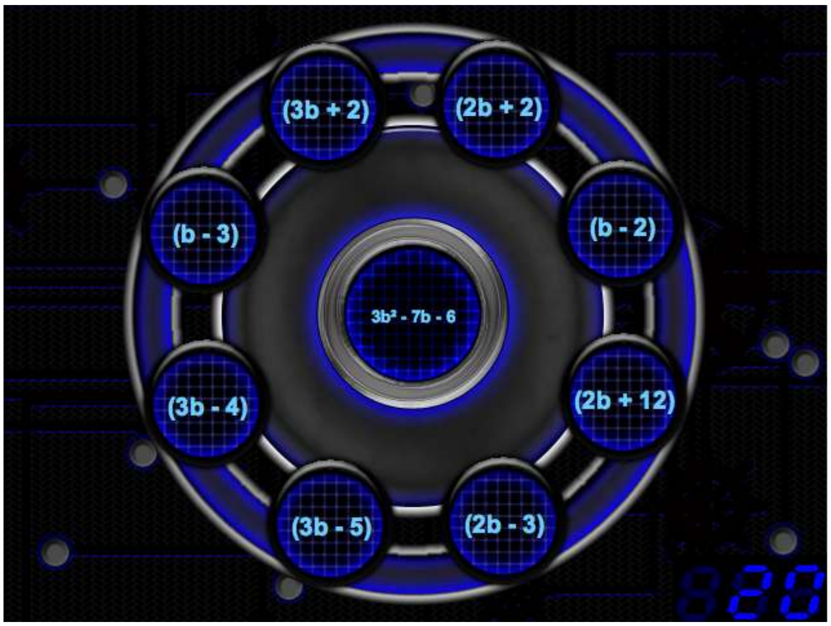


Figure 1. Match Factors players choose the factors of a central polynomial from a rotating ring of possibilities.

Description

Each *Match Factors* game consists of a sequence of 4 polynomials surrounded by 8 possible factors. As each polynomial is displayed, players are asked to pick the matching pair of factors. If they choose correctly, the next polynomial appears. If not, they must try again before play advances.

There are three levels of play, each containing 10 games. In Level 1, polynomials have the form $x^2 + bx + c$. Level 2 polynomials have the form $ax^2 + bx + c$. Players in Level 3 must factor $ax^2 + bxy + cy^2$ polynomials.

Match Factors is suitable for individual or group play. It could also be used in a classroom setting, with the whole group taking turns calling out the two factors of each expression.

Unit 12: Factoring

Instructor Overview
Project: Making Connections

Student Instructions

Introduction

The main business of science is to uncover patterns. Often we represent those patterns as algebraic expressions, graphs, or tables of numbers (data). Being able to make connections among those various representations is one of the most vital skills to possess.

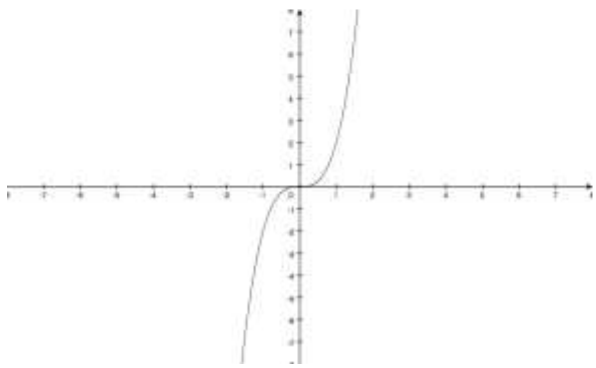
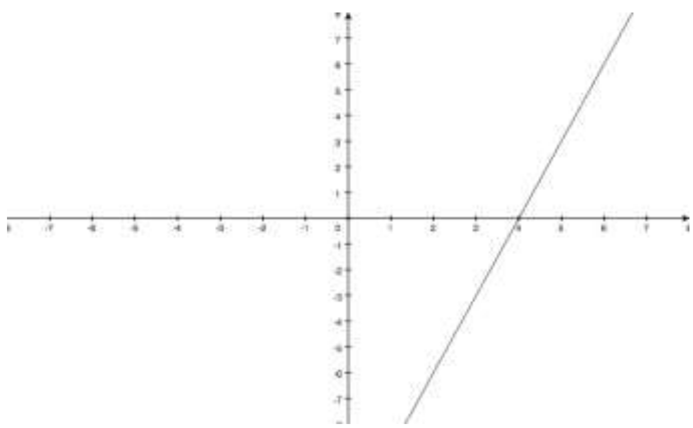
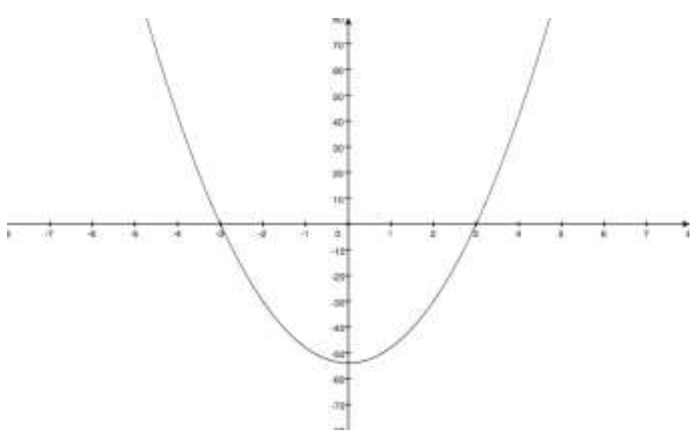
Task

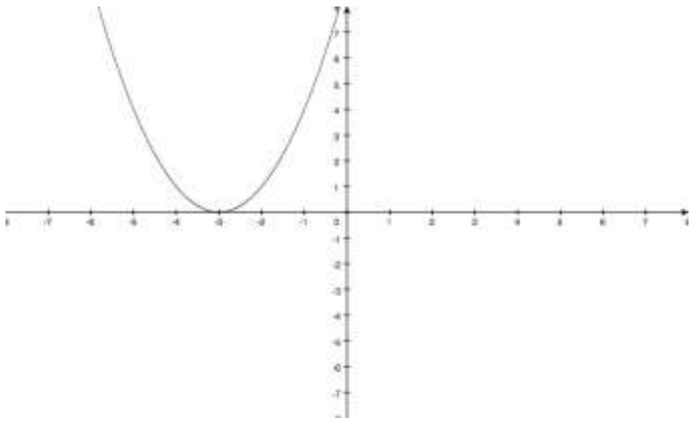
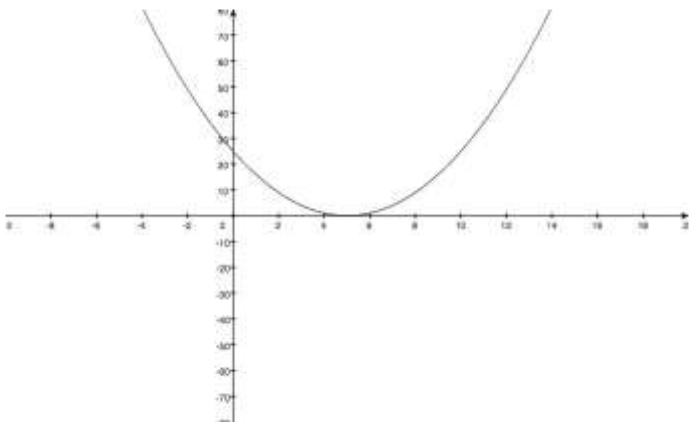
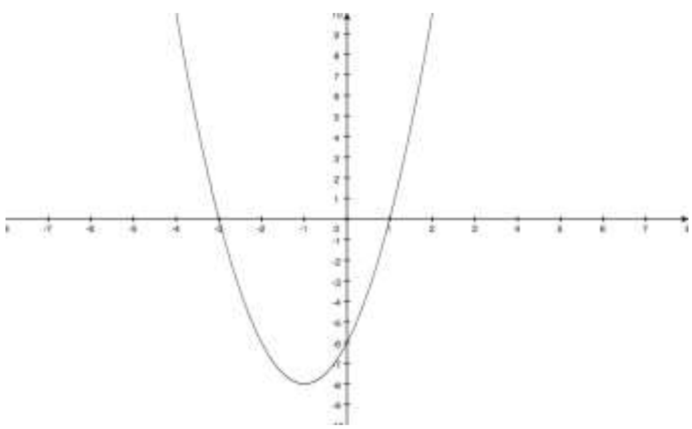
In this project you attempt to make precise connections among these three ways of representing patterns.

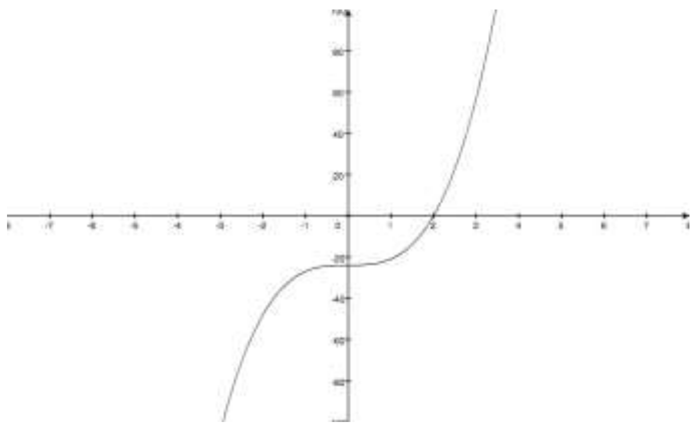
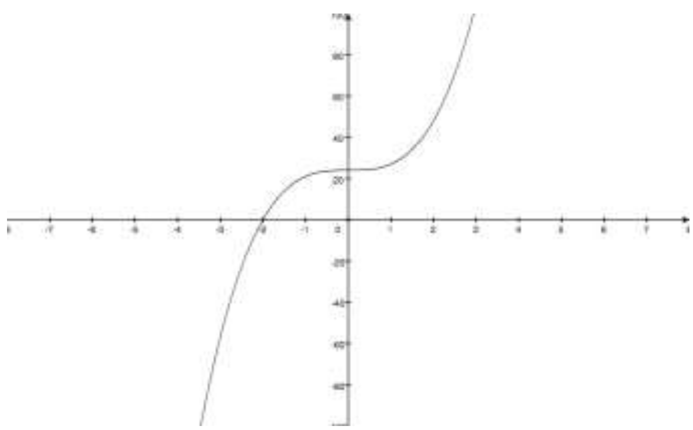
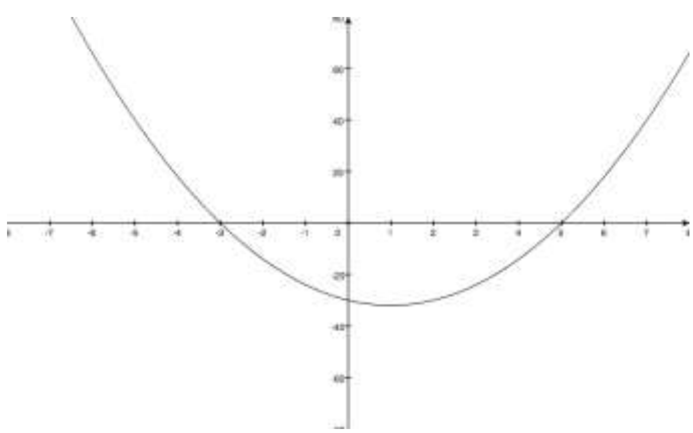
Instructions (See the online course materials for full size graphs and charts)

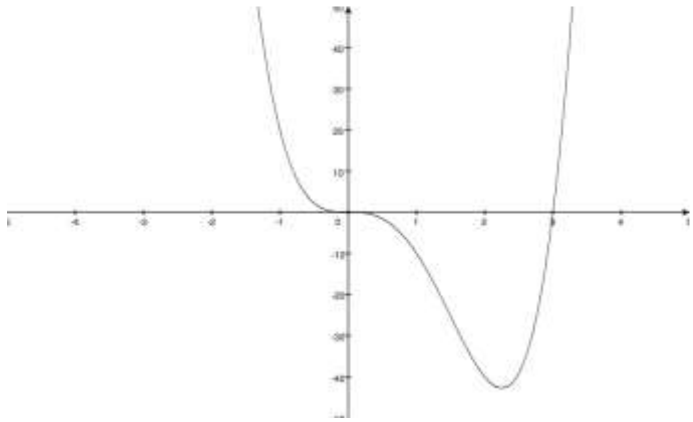
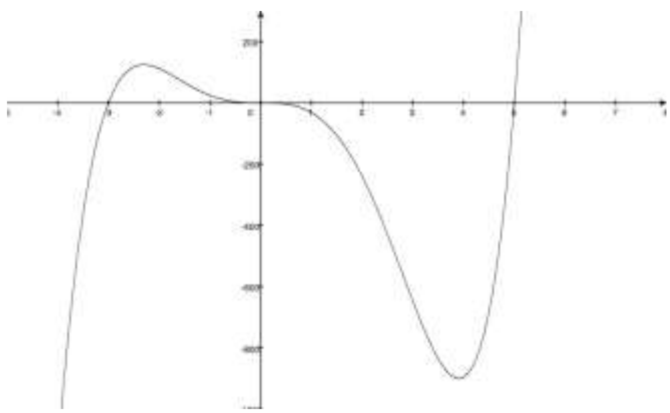
Work with at least one other person to complete the following exercises. Solve each problem in order and save your work along the way. You will create a presentation on one of the four parts to be given to your class.

- **First Problem – Connecting Algebraic Expressions and Graphs:** Factor each of the following expressions completely, and then compare the factored form with the "picture" of the expression that is shown in the graph on the right. Describe any connections that you see, and then repeat for the next expression. In the end, formulate a conjecture as to how an algebraic expression in factored form is related to its corresponding graph. Keep in mind that we are not assuming that you have any knowledge whatsoever about graphs. That is what makes this "detective work" so fun!

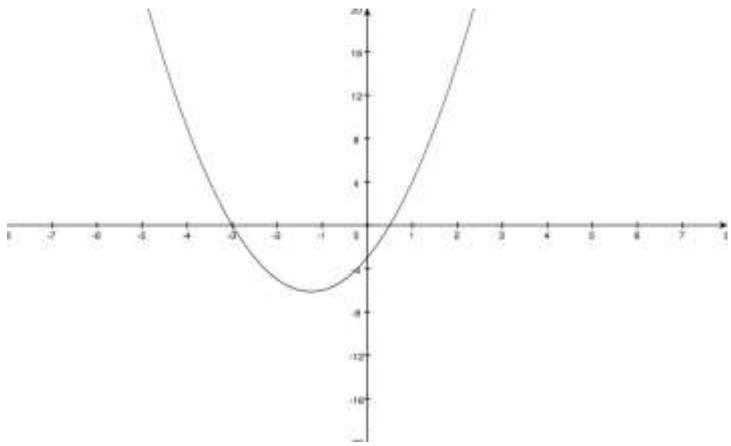
Algebraic Expression	**Graph**	**Possible Relationship**
$2a^3$ **Factored Form**		
$3b-12$ **Factored Form**		
$6x^2-54$ **Factored Form**		

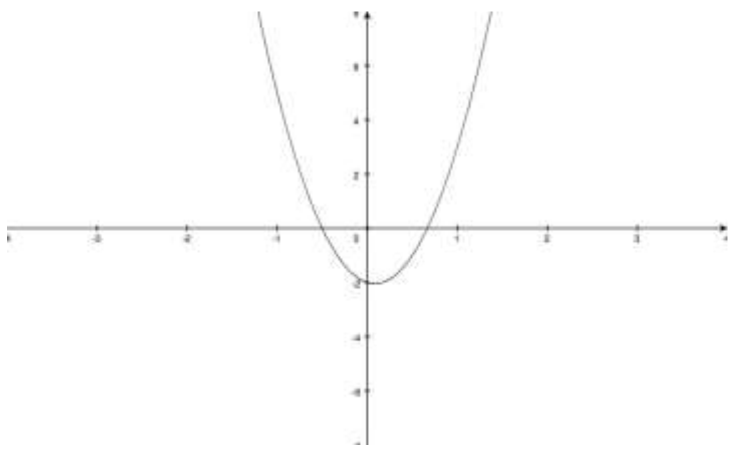
Algebraic Expression	Graph	Possible Relationship
$x^2 + 6x + 9$		
Factored Form		
$y^2 - 10y + 25$		
Factored Form		
$2c^2 + 4c - 6$		
Factored Form		

Algebraic Expression	**Graph**	**Possible Relationship**
$3a^3 - 24$ **Factored Form**		
$3a^3 + 24$ **Factored Form**		
$2x^2 - 4x - 30$ **Factored Form**		

Algebraic Expression	Graph	Possible Relationship
$5d^4 - 15d^3$		
Factored Form		
$2x^5 - 4x^4 - 30x^3$		
Factored Form		

<table>
<tr><td colspan="3">The following problems are a special challenge. You may have to adjust the relationship you expressed above to accommodate these new examples.</td></tr>
<tr><td>$2x^2+5x-3$

Factored Form</td><td></td><td></td></tr>
<tr><td>$6t^2-t-2$

Factored Form</td><td></td><td></td></tr>
</table>

- **Second Problem – Connecting Algebraic Expressions and Tables:** For each of the same algebraic expressions that you examined above, compare the factored form with the table of values associated with the expression. (For example, if the expression is $2x^2+5x-3$, then the value associated with it when x=1 will be $2(1)^2+5(1)-3=4$.) In the end, formulate a conjecture that describes how an algebraic expression in factored form is related to its corresponding data table.

<table>
<tr><th>Algebraic Expression</th><th>Table</th><th>Possible Relationship</th></tr>
<tr><td>$2a^3$

Factored Form</td><td>

a	$2a^3$
-8	-1024
-7	-686
-6	-432
-5	-250
-4	-128
-3	-54
-2	-16
-1	-2
0	0
1	2
2	16
3	54
4	128
5	250
6	432
7	686
8	1024

</td><td></td></tr>
<tr><td>$3b-12$</td><td>

b	$3b-12$
-8	-36
-7	-33
-6	-30

</td><td></td></tr>
</table>

<table>
<tr><th>Algebraic Expression</th><th>Table</th><th>Possible Relationship</th></tr>
<tr><td>Factored Form</td><td>
<table>
<tr><td>-5</td><td>-27</td></tr>
<tr><td>-4</td><td>-24</td></tr>
<tr><td>-3</td><td>-21</td></tr>
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<table>
<tr><th>Algebraic Expression</th><th>Table</th><th>Possible Relationship</th></tr>
<tr><td>Factored Form</td><td>

-5	4
-4	1
-3	0
-2	1
-1	4
0	9
1	16
2	25
3	36
4	49
5	64
6	81
7	100
8	121

</td><td></td></tr>
<tr><td>$y^2 - 10y + 25$</td><td>

y	$y^2 - 10y + 25$
-8	169
-7	144
-6	121

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</table>

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<tr><th>Algebraic Expression</th><th>Table</th><th>Possible Relationship</th></tr>
<tr><td>Factored Form</td><td>
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<tr><td>-5</td><td>100</td></tr>
<tr><td>-4</td><td>81</td></tr>
<tr><td>-3</td><td>64</td></tr>
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<tr><td>$2c^2 + 4c - 6$</td><td>
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<tr><td>-8</td><td>90</td></tr>
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<tr><th>Algebraic Expression</th><th>Table</th><th>Possible Relationship</th></tr>
<tr><td>Factored Form</td><td>
<table>
<tr><td>-5</td><td>24</td></tr>
<tr><td>-4</td><td>10</td></tr>
<tr><td>-3</td><td>0</td></tr>
<tr><td>-2</td><td>-6</td></tr>
<tr><td>-1</td><td>-8</td></tr>
<tr><td>0</td><td>-6</td></tr>
<tr><td>1</td><td>0</td></tr>
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<tr><td>$3a^3 - 24$</td><td>
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<tr><td>a</td><td>$3a^3 - 24$</td></tr>
<tr><td>-8</td><td>-1560</td></tr>
<tr><td>-7</td><td>-1053</td></tr>
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<tr><th>Algebraic Expression</th><th>Table</th><th>Possible Relationship</th></tr>
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<tr><td>-4</td><td>-216</td></tr>
<tr><td>-3</td><td>-105</td></tr>
<tr><td>-2</td><td>-48</td></tr>
<tr><td>-1</td><td>-27</td></tr>
<tr><td>0</td><td>-24</td></tr>
<tr><td>1</td><td>-21</td></tr>
<tr><td>2</td><td>0</td></tr>
<tr><td>3</td><td>57</td></tr>
<tr><td>4</td><td>168</td></tr>
<tr><td>5</td><td>351</td></tr>
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<tr><td>-6</td><td>-624</td></tr>
</table>
</td><td></td></tr>
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Algebraic Expression	Table	Possible Relationship
Factored Form	-5	-351 -4	-168 -3	-57 -2	0 -1	21 0	24 1	27 2	48 3	105 4	216 5	399 6	672 7	1053 8	1560	
$2x^2 - 4x - 30$	x	$2x^2 - 4x - 30$ -8	130 -7	96 -6	66	

<table>
<tr><th>Algebraic Expression</th><th>Table</th><th>Possible Relationship</th></tr>
<tr><td>Factored Form</td><td>
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<tr><td>-5</td><td>40</td></tr>
<tr><td>-4</td><td>18</td></tr>
<tr><td>-3</td><td>0</td></tr>
<tr><td>-2</td><td>-14</td></tr>
<tr><td>-1</td><td>-24</td></tr>
<tr><td>0</td><td>-30</td></tr>
<tr><td>1</td><td>-32</td></tr>
<tr><td>2</td><td>-30</td></tr>
<tr><td>3</td><td>-24</td></tr>
<tr><td>4</td><td>-14</td></tr>
<tr><td>5</td><td>0</td></tr>
<tr><td>6</td><td>18</td></tr>
<tr><td>7</td><td>40</td></tr>
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<tr><td>-7</td><td>17150</td></tr>
<tr><td>-6</td><td>9720</td></tr>
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Algebraic Expression	Table	Possible Relationship
Factored Form	<table><tr><td>-5</td><td>5000</td></tr><tr><td>-4</td><td>2240</td></tr><tr><td>-3</td><td>810</td></tr><tr><td>-2</td><td>200</td></tr><tr><td>-1</td><td>20</td></tr><tr><td>0</td><td>0</td></tr><tr><td>1</td><td>-10</td></tr><tr><td>2</td><td>-40</td></tr><tr><td>3</td><td>0</td></tr><tr><td>4</td><td>320</td></tr><tr><td>5</td><td>1250</td></tr><tr><td>6</td><td>3240</td></tr><tr><td>7</td><td>6860</td></tr><tr><td>8</td><td>12800</td></tr></table>	
$2x^5 - 4x^4 - 30x^3$	<table><tr><td>x</td><td>$2x^5 - 4x^4 - 30x^3$</td></tr><tr><td>-8</td><td>-66560</td></tr><tr><td>-7</td><td>-32928</td></tr></table>	

<table>
<tr><th>Algebraic Expression</th><th>Table</th><th>Possible Relationship</th></tr>
<tr><td>Factored Form</td><td>
<table>
<tr><td>-6</td><td>-14256</td></tr>
<tr><td>-5</td><td>-5000</td></tr>
<tr><td>-4</td><td>-1152</td></tr>
<tr><td>-3</td><td>0</td></tr>
<tr><td>-2</td><td>112</td></tr>
<tr><td>-1</td><td>24</td></tr>
<tr><td>0</td><td>0</td></tr>
<tr><td>1</td><td>-32</td></tr>
<tr><td>2</td><td>-240</td></tr>
<tr><td>3</td><td>-648</td></tr>
<tr><td>4</td><td>-896</td></tr>
<tr><td>5</td><td>0</td></tr>
<tr><td>6</td><td>3888</td></tr>
<tr><td>7</td><td>13720</td></tr>
<tr><td>8</td><td>33792</td></tr>
</table>
</td><td></td></tr>
<tr><td>$2x^2+5x-3$</td><td>
<table>
<tr><td>x</td><td>$2x^2+5x-3$</td></tr>
<tr><td>-8</td><td>85</td></tr>
<tr><td>-7</td><td>60</td></tr>
<tr><td>-6</td><td>39</td></tr>
</table>
</td><td></td></tr>
</table>

<table>
<tr><th>Algebraic Expression</th><th>Table</th><th>Possible Relationship</th></tr>
<tr><td>Factored Form</td><td>
<table>
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<tr><td>-4</td><td>9</td></tr>
<tr><td>-3</td><td>0</td></tr>
<tr><td>-2</td><td>-5</td></tr>
<tr><td>-1</td><td>-6</td></tr>
<tr><td>0</td><td>-3</td></tr>
<tr><td>1</td><td>4</td></tr>
<tr><td>2</td><td>15</td></tr>
<tr><td>3</td><td>30</td></tr>
<tr><td>4</td><td>49</td></tr>
<tr><td>5</td><td>72</td></tr>
<tr><td>6</td><td>99</td></tr>
<tr><td>7</td><td>130</td></tr>
<tr><td>8</td><td>165</td></tr>
</table>
</td><td></td></tr>
<tr><td>$6t^2 - t - 2$</td><td>
<table>
<tr><td>t</td><td>$6t^2 - t - 2$</td></tr>
<tr><td>-8</td><td>390</td></tr>
<tr><td>-7</td><td>299</td></tr>
<tr><td>-6</td><td>220</td></tr>
</table>
</td><td></td></tr>
</table>

Algebraic Expression	Table		Possible Relationship
Factored Form	-5	153	
	-4	98	
	-3	55	
	-2	24	
	-1	5	
	0	-2	
	1	3	
	2	20	
	3	49	
	4	90	
	5	143	
	6	208	
	7	285	
	8	374	

- **Third Problem – Applying Your Findings:** For each expression, factor it completely and write the factored form beneath the expression. Then match it to its corresponding table or graph by writing the letter corresponding to the expression on its matching table or graph.

a) $3c - 12$

b) $2a^5 - 9a^4$

c) $3d^5 - 12d^3$

d) $y^3 - 27$

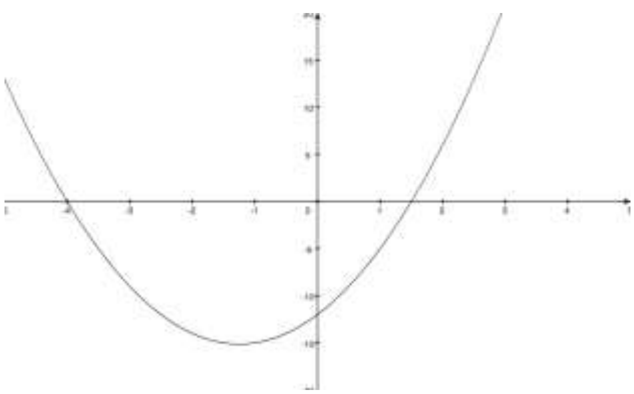
e) $2z^3 - 16$

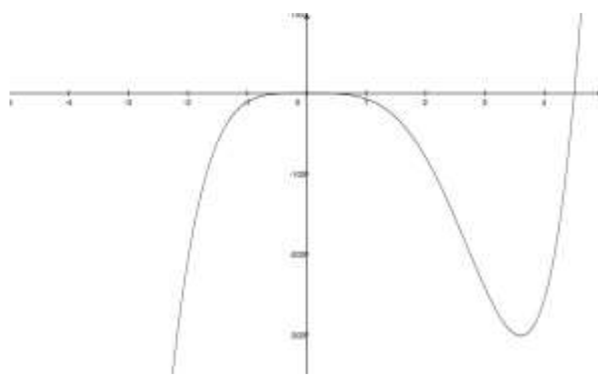
f) $2x^2+5x-12$

g) $3a^7$

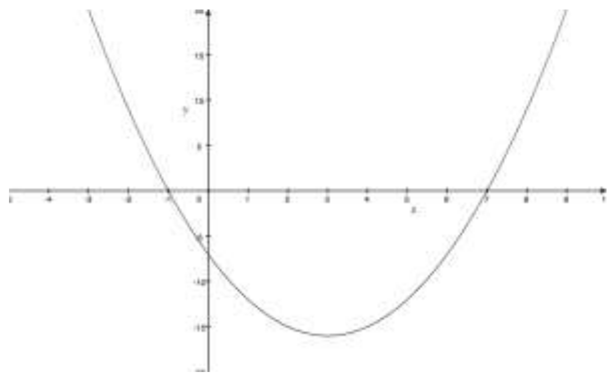
h) $t^2 - 6t - 7$

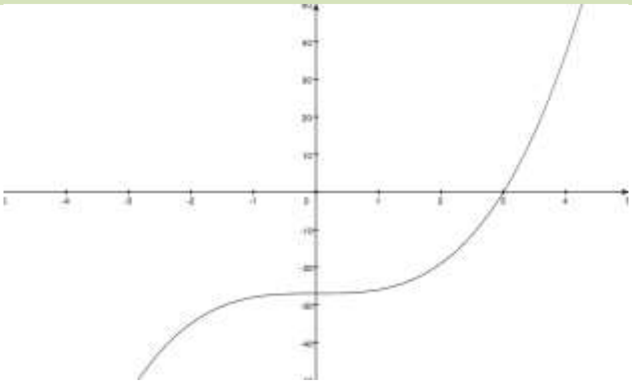
i) $9y^2 + 6y + 1$

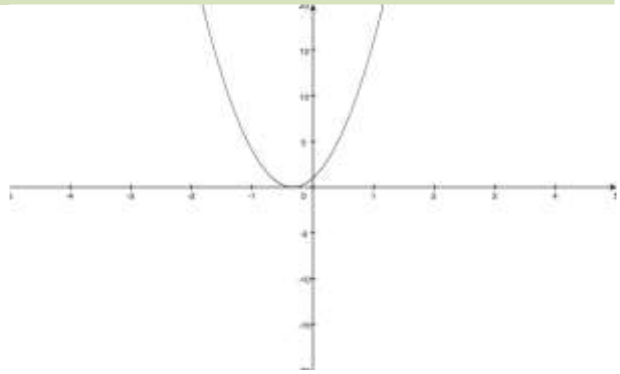




Variable	Expression
-8	-92160
-7	-46305
-6	-20736
-5	-7875
-4	-2304
-3	-405
-2	0
-1	9
0	0
1	-9
2	0
3	405
4	2304
5	7875
6	20736
7	46305
8	92160





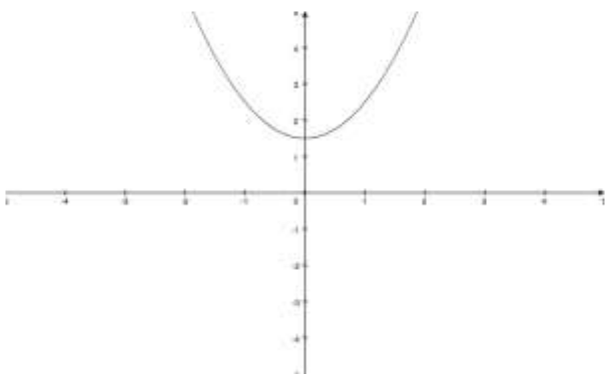


Variable	Expression
-8	-6291456
-7	-2470629
-6	-839808
-5	-234375
-4	-49152
-3	-6561
-2	-384
-1	-3
0	0
1	3
2	384
3	6561
4	49152
5	234375
6	839808
7	2470629
8	6291456

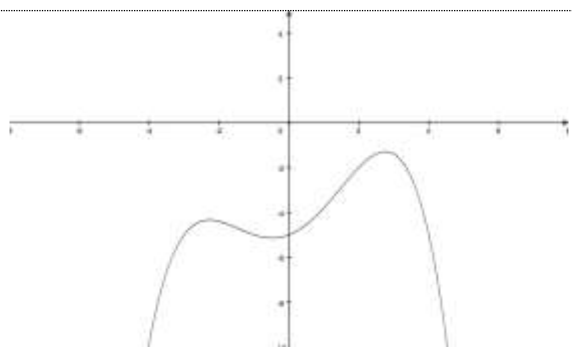
Variable	Expression
-8	-1040
-7	-702
-6	-448
-5	-266
-4	-144
-3	-70
-2	-32
-1	-18
0	-16
1	-14
2	0
3	38
4	112
5	234
6	416
7	670
8	1008

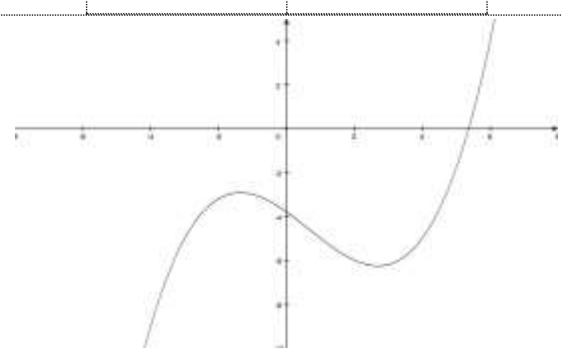
Variable	Expression
-8	-36
-7	-33
-6	-30
-5	-27
-4	-24
-3	-21
-2	-18
-1	-15
0	-12
1	-9
2	-6
3	-3
4	0
5	3
6	6
7	9
8	12

- **Fourth Problem – Predicting the Unknown:** One of the primary reasons to make connections is to be able to explain or predict previously unobserved behavior. Below we provide you with some tables and some graphs. Based on these alone, determine whether the expression associated with them can be factored. Explain the reasoning behind your decision. *[Hint: You should make use of your observations from the problems above to determine what it means for an expression to not be factorable.]*



Variable	Expression
-8	-550
-7	-376
-6	-244
-5	-148
-4	-82
-3	-40
-2	-16
-1	-4
0	2
1	8
2	20
3	44
4	86
5	152
6	248
7	380
8	554





Variable	Expression
-8	344
-7	193
-6	98
-5	44
-4	18
-3	8
-2	7
-1	8
0	9
1	6
2	2
3	1
4	8
5	32
6	84
7	176
8	325

Conclusions

With those from another group, compare your answers and your way of talking about the connections between the factored form of the expressions and the graphs and tables. Work to make sure that your explanation is clear and concise.

Prepare a presentation which:

1. Explains the connection between the factored expression and the corresponding graphs and tables.

2. Describes briefly how you determined this connection (you may want to discuss some of your original ideas and how you needed to refine them as you looked at more examples).
3. Gives a test for determining whether a given expression can be factored if you are given a graph or table associated with the expression.

Finally, present your solution to your instructor.

Instructor Notes

We would stress that nothing in this project assumes that students have any familiarity whatsoever with graphing. In fact, it is precisely because they do not have this familiarity that we can explore this topic. The project is more about developing students' abilities to notice connections between the various functional representations before they even understand how these representations work. So, this project can serve both as a culminating project for Unit 12 on factoring as well as very initial preparation for a unit on graphing.

Assignment Procedures

Problem 1

The relationship that they should be identifying is that the values of the variable that make each factor zero will correspond to the places where the graph crosses the horizontal axis. For the first eleven graphs, the student may very well identify the "the negative of the number in the factor" as the place where the graph crosses the axis. For example, for $2c^2 + 4c - 6 = 2 \cdot (c+3) \cdot (c-1)$, the graph crosses at $c=-3$ and $c=1$. However, when they encounter the last two, for example $2x^2 + 5x - 3 = (2x-1) \cdot (x+3)$, the graph does indeed cross at $x=-3$, but it crosses a second time at $x=\frac{1}{2}$ and not at $x=1$, even though $x=1$ is the "negative of the number that appears in the factor." It may be a challenge for them to determine the true connection, although the fact that they have had experience solving quadratic equations by factoring should facilitate the process.

Problem 2

The connection is that the value of the variable which makes each factor equal to zero (and therefore the entire expression equal to zero as well) is the one which corresponds to a zero value for the expression in the table. Notice that, since the values of the variable rise in increments of one, the exact values of the variable that correspond to a zero value for the expression do not actually appear in the last two tables. In these examples, the students will have to notice that the value of the expression changes sign and, therefore, must have been zero somewhere in between.

Problem 3

The following table shows the correct matching.

Graph: f	Graph: b
Table: c	Graph: h
Graph: d	Graph: i
Table: g	Table: e
Table: a	

Problem 4

The table below shows the solutions.

Graph: not factorable since the graph does not cross the horizontal axis.	**Table:** factorable because somewhere between x=-1 and x=0 it is equal to zero.
Graph: not factorable since the graph does not cross the horizontal axis.	**Graph:** factorable since the graph crosses the horizontal axis somewhere between 4 and 6.
Table: not factorable since nowhere does it appear that the expression changes sign or is zero. Note that this is only speculative since the table shows data only for values of x that are integers. However, the students at this stage need not be attentive to this nuance.	

At this stage it can be helpful to tell the students in each group that for each question, you will randomly choose one person in the group to present the group's answer. This provides motivation for the group as a whole to ensure that each member has a thorough understanding of all of the topics and gives the instructor feedback on how well each individual understands the work that was completed.

Recommendations

- Have students work in teams to encourage brainstorming and cooperative learning.
- Assign a specific timeline for completion of the project that includes milestone dates.
- Provide students feedback as they complete each milestone.
- Ensure that each member of student groups has a specific job.

Technology Integration

This project provides abundant opportunities for technology integration, and gives students the chance to research and collaborate using online technology. The students' instructions list several websites that provide information on numbering systems, game design, and graphics.

The following are other examples of free Internet resources that can be used to support this project:

http://www.moodle.org

An Open Source Course Management System (CMS), also known as a Learning Management System (LMS) or a Virtual Learning Environment (VLE). Moodle has become very popular among educators around the world as a tool for creating online dynamic websites for their students.

http://www.wikispaces.com/site/for/teachers or http://pbworks.com/content/edu+overview

Allows you to create a secure online Wiki workspace in about 60 seconds. Encourage classroom participation with interactive Wiki pages that students can view and edit from any computer. Share class resources and completed student work.

http://www.docs.google.com

Allows students to collaborate in real-time from any computer. Google Docs provides free access and storage for word processing, spreadsheets, presentations, and surveys. This is ideal for group projects.

http://why.openoffice.org/

The leading open-source office software suite for word processing, spreadsheets, presentations, graphics, databases and more. It can read and write files from other common office software packages like Microsoft Word or Excel and MacWorks. It can be downloaded and used completely free of charge for any purpose.

Rubric

Score	Content	Presentation/Communication
4	• The solution shows a deep understanding of the problem including the ability to identify the appropriate mathematical concepts and the information necessary for its solution. • The solution completely addresses all mathematical components presented in the task. • The solution puts to use the underlying mathematical concepts upon which the task is designed and applies procedures accurately to correctly solve the problem and verify the results. • Mathematically relevant observations and/or connections are made.	• There is a clear, effective explanation detailing how the problem is solved. All of the steps are included so that the reader does not need to infer how and why decisions were made. • Mathematical representation is actively used as a means of communicating ideas related to the solution of the problem. • There is precise and appropriate use of mathematical terminology and notation. • Your project is professional looking with graphics and effective use of color.
3	• The solution shows that the student has a broad understanding of the problem and the major concepts necessary for its solution. • The solution addresses all of the mathematical components presented in the task. • The student uses a strategy that includes mathematical procedures and some mathematical reasoning that leads to a solution of the problem. • Most parts of the project are correct with only minor mathematical errors.	• There is a clear explanation. • There is appropriate use of accurate mathematical representation. • There is effective use of mathematical terminology and notation. • Your project is neat with graphics and effective use of color.
2	• The solution is not complete indicating that parts of the problem are not understood. • The solution addresses some, but not all of the mathematical components presented in the task. • The student uses a strategy that is partially useful, and demonstrates some evidence of mathematical reasoning. • Some parts of the project may be correct, but major errors are noted and the student could not completely carry out mathematical procedures.	• Your project is hard to follow because the material is presented in a manner that jumps around between unconnected topics. • There is some use of appropriate mathematical representation. • There is some use of mathematical terminology and notation appropriate to the problem. • Your project contains low quality graphics and colors that do not add interest to the project.
1	• There is no solution, or the solution has no relationship to the task. • No evidence of a strategy, procedure, or mathematical reasoning and/or uses a strategy that does not help solve the problem.	• There is no explanation of the solution, the explanation cannot be understood or it is unrelated to the problem. • There is no use or inappropriate use of mathematical representations (e.g.

	• The solution addresses none of the mathematical components presented in the task. • There were so many errors in mathematical procedures that the problem could not be solved.	figures, diagrams, graphs, tables, etc.). • There is no use, or mostly inappropriate use, of mathematical terminology and notation. • Your project is missing graphics and uses little to no color.

Unit 12 – Correlation to Common Core Standards

Unit 12: Factoring

Common Core Standards

Unit 12, Lesson 1, Topic 1: Greatest Common Factor

Grade: **8** - Adopted **2010**

STRAND / DOMAIN	**CC.MP.**	Mathematical Practices
CATEGORY / CLUSTER	**MP.1.**	Make sense of problems and persevere in solving them.

Grade: **9-12** - Adopted **2010**

STRAND / DOMAIN	**CC.MP.**	Mathematical Practices
CATEGORY / CLUSTER	**MP.1.**	Make sense of problems and persevere in solving them.
STRAND / DOMAIN	**CC.A.**	Algebra
CATEGORY / CLUSTER	**A-SSE.**	Seeing Structure in Expressions
STANDARD		Interpret the structure of expressions.
EXPECTATION	**A-SSE.1.**	Interpret expressions that represent a quantity in terms of its context.
GRADE EXPECTATION	**A-SSE.1(a)**	Interpret parts of an expression, such as terms, factors, and coefficients.

Unit 12, Lesson 2, Topic 1: Factor Trinomials

Grade: **8** - Adopted **2010**

STRAND / DOMAIN	**CC.MP.**	Mathematical Practices
CATEGORY / CLUSTER	**MP.1.**	Make sense of problems and persevere in solving them.

Grade: **9-12** - Adopted **2010**

STRAND / DOMAIN	**CC.MP.**	Mathematical Practices
CATEGORY / CLUSTER	**MP.1.**	Make sense of problems and persevere in solving them.
STRAND / DOMAIN	**CC.A.**	Algebra
CATEGORY / CLUSTER	**A-SSE.**	Seeing Structure in Expressions
STANDARD		Interpret the structure of expressions.
EXPECTATION	**A-SSE.1.**	Interpret expressions that represent a quantity in terms of its context.
GRADE EXPECTATION	**A-SSE.1(a)**	Interpret parts of an expression, such as terms, factors, and coefficients.

Unit 12, Lesson 2, Topic 2: Factoring: Special Cases

Grade: **8** - Adopted **2010**

STRAND / DOMAIN	**CC.MP.**	Mathematical Practices
CATEGORY / CLUSTER	**MP.1.**	Make sense of problems and persevere in solving them.

Grade: **9-12** - Adopted **2010**

STRAND / DOMAIN	**CC.MP.**	Mathematical Practices
CATEGORY / CLUSTER	**MP.1.**	Make sense of problems and persevere in solving them.
STRAND / DOMAIN	**CC.A.**	Algebra
CATEGORY / CLUSTER	**A-SSE.**	Seeing Structure in Expressions
STANDARD		Interpret the structure of expressions.
EXPECTATION	**A-SSE.1.**	Interpret expressions that represent a quantity in terms of its context.
GRADE EXPECTATION	**A-SSE.1(a)**	Interpret parts of an expression, such as terms, factors, and coefficients.

Unit 12, Lesson 2, Topic 3: Special Cases: Cubes

Grade: **8** - Adopted **2010**

STRAND / DOMAIN	**CC.MP.**	Mathematical Practices
CATEGORY / CLUSTER	**MP.1.**	Make sense of problems and persevere in solving them.

Grade: **9-12** - Adopted **2010**

STRAND / DOMAIN	**CC.MP.**	Mathematical Practices
CATEGORY / CLUSTER	**MP.1.**	Make sense of problems and persevere in solving them.
STRAND / DOMAIN	**CC.A.**	Algebra
CATEGORY / CLUSTER	**A-SSE.**	Seeing Structure in Expressions
STANDARD		Interpret the structure of expressions.
EXPECTATION	**A-SSE.1.**	Interpret expressions that represent a quantity in terms of its context.
GRADE EXPECTATION	**A-SSE.1(a)**	Interpret parts of an expression, such as terms, factors, and coefficients.

Unit 12, Lesson 3, Topic 1: Solve Quadratic Equations by Factoring

Grade: **8** - Adopted **2010**

STRAND / DOMAIN	**CC.MP.**	Mathematical Practices
CATEGORY / CLUSTER	**MP.1.**	Make sense of problems and persevere in solving them.

Grade: **9-12** - Adopted **2010**

STRAND / DOMAIN	**CC.MP.**	Mathematical Practices
CATEGORY / CLUSTER	**MP.1.**	Make sense of problems and persevere in solving them.
STRAND / DOMAIN	**CC.A.**	Algebra

CATEGORY / CLUSTER	**A-SSE.**	Seeing Structure in Expressions
STANDARD		Write expressions in equivalent forms to solve problems.
EXPECTATION	**A-SSE.3.**	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
GRADE EXPECTATION	**A-SSE.3(a)**	Factor a quadratic expression to reveal the zeros of the function it defines.
STRAND / DOMAIN	**CC.A.**	Algebra
CATEGORY / CLUSTER	**A-REI.**	Reasoning with Equations and Inequalities
STANDARD		Solve equations and inequalities in one variable.
EXPECTATION	**A-REI.4.**	Solve quadratic equations in one variable.
GRADE EXPECTATION	**A-REI.4(b)**	Solve quadratic equations by inspection (e.g., for x^2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a plus-minus bi for real numbers a and b.
STRAND / DOMAIN	**CC.F.**	Functions
CATEGORY / CLUSTER	**F-IF.**	Interpreting Functions
STANDARD		Analyze functions using different representations.
EXPECTATION	**F-IF.8.**	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
GRADE EXPECTATION	**F-IF.8(a)**	Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.