Unit 9: Factoring

Unit 9: Factoring

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Unit 9: Factoring

Instructor Notes

The Mathematics of Factoring Polynomials

This unit builds upon students' knowledge of factoring to teach them how to pull apart polynomials. They will learn how to use the distributive property and greatest common factors to find the factored form of binomials and how to factor trinomials with grouping. Students will also learn how to recognize and quickly factor special products (perfect square trinomials and difference of squares binomials). Finally, they'll get experience combining these techniques by using them to solve quadratic equations.

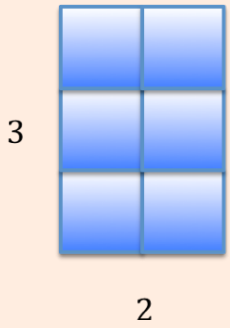
Teaching Tips: Conceptual Challenges and Approaches

This unit on factoring is probably one of the most difficult—students will spend a lot of time carrying out multi-step, complex procedures for what will often seem to be obscure purposes. At this stage in algebra, factoring polynomials may feel like busy work rather than a means to a useful end. To combat the high level of abstraction, we begin by connecting the new mathematics to ideas students are familiar with, prime and composite numbers. Visual models and manipulatives help extend these concepts to polynomials.

Topic 1 reviews the mathematics involved in finding greatest common factors before demonstrating how to factor expressions by using the distributive property in reverse to pull out the greatest common monomial from each term in a polynomial.

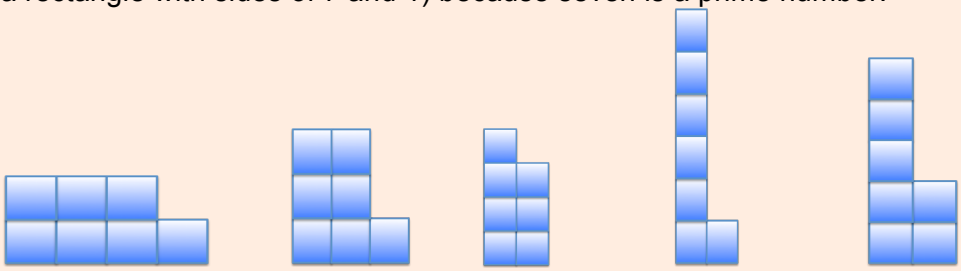
If students are struggling with these ideas then it may be valuable to revisit the area model of multiplication. A graphical representation of prime and composite numbers can help students understand how to apply the same ideas to trinomials.

Example
The number six, which is a composite number, can be represented as the area of a rectangle like this:



Students can clearly see that 3 and 2 are factors of 6.

In contrast, it is impossible to make a rectangle out of seven tiles (other than the case of a rectangle with sides of 7 and 1) because seven is a prime number:



This visual representation, used in Unit 8, can be continued to support the work with factoring trinomials.

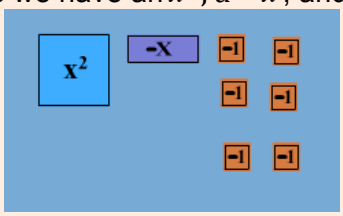
The example above used the area model as a basis for understanding prime and composite numbers. Students can use similar manipulatives to model the area representation of trinomials. An excellent example can be found here [MAC users will need to copy/paste url into browser]:

http://courses.wccnet.edu/~rwhatcher/VAT/Factoring/

Having students work with a manipulative like this as they begin to build their understanding, either individually or in small groups, could help all students, including English Learners, build their conceptual understanding.

Example

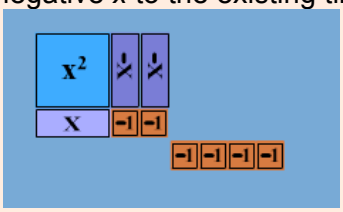
One key mathematical point to raise concerns building rectangles to represent trinomials such as $x^2 - x - 6$. In this case we have an x^2, a $-x$, and -6.



$x^2 - x - 6$

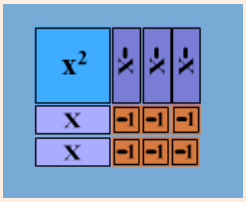
Notice that it is impossible to build a rectangle from these pieces, even though this is a factorable trinomial, with factors $(x - 3)$ and $(x + 2)$. Explain to students that we can fill in the rectangle by adding more pieces, as long as they don't change the value of the expression. For example, pairs of +*x* and –*x* will help build the rectangle but they cancel each other out in the polynomial.

So let's add a positive *x* and a negative *x* to the existing tiles:



$x^2 - 2x + x - 6$

We're not there yet, but you can see we're getting closer, so let's add another pair, +*x* and –*x*,



$x^2 - 3x + 2x - 6$

And now we have a rectangle.

Teaching Tips: Algorithmic Challenges and Approaches

The process shown in the example above exactly models factoring by grouping. This is the technique taught in Topics 2 and 3.

Many textbooks do not use factoring by grouping for trinomials, and instead use essentially a guess and check method. While factoring by grouping may initially be a more complex procedure, it has many significant advantages in the long term for students and is used in this course.

Factoring by grouping has the great advantage of working for all trinomials, from the most basic all the way to simple cubic polynomials, for example:

$$x^3 + 3x^2 - 4x - 12$$
$$x^2(x + 3) - 4(x + 3)$$
$$(x^2 - 4)(x + 3)$$
$$(x - 2)(x + 2)(x + 3)$$

Students are now equipped with a powerful technique that will transfer into Algebra 2 and other mathematics courses.

Summary

Factoring trinomials and solving quadratic equations by factoring are some of the most abstract mathematics in Algebra 1, and students will struggle to learn the techniques because they don't see the point. Connecting these procedures to prior ideas of prime and composite numbers, and using a powerful technique like factoring by grouping, which works in all cases, will help students confidently work with this complex mathematics.

Unit 9: Factoring

Instructor Overview
Tutor Simulation: Perfecting the Long Kick in Soccer

Purpose

This simulation is designed to challenge a student's understanding of factoring polynomials. Students will be asked to apply what they have learned to solve a real world problem by demonstrating understanding of the following areas:

- Visualizing Problems
- Factoring Polynomials
- Trajectories

Problem

Students are given the following problem:

> Your school's soccer coach spent a lot of money on a computer system that would analyze his team's kicking technique, but the system broke down before he got the results for his star player, Renaldo.
>
> It's up to you to take over and finish the trajectory calculations.

Recommendations

Tutor simulations are designed to give students a chance to assess their understanding of unit material in a personal, risk-free situation. Before directing students to the simulation,

- make sure they have completed all other unit material.
- explain the mechanics of tutor simulations
  - Students will be given a problem and then guided through its solution by a video tutor;
  - After each answer is chosen, students should wait for tutor feedback before continuing;
  - After the simulation is completed, students will be given an assessment of their efforts. If areas of concern are found, the students should review unit materials or seek help from their instructor.
- emphasize that this is an exploration, not an exam.

Unit 9: Factoring

Instructor Overview
Puzzle: Match Factors

Objective

Match Factors is a puzzle that tests a player's ability to factor trinomials and provides a fun means of practicing grouping. It reinforces the technique of factoring a trinomial in the form $ax^2 + bx + c$ by finding two integers, r and s, whose sum is b and whose product is ac. Puzzle play, especially when done by eye rather than with pencil and paper, will help students learn to quickly identify the components of factors.

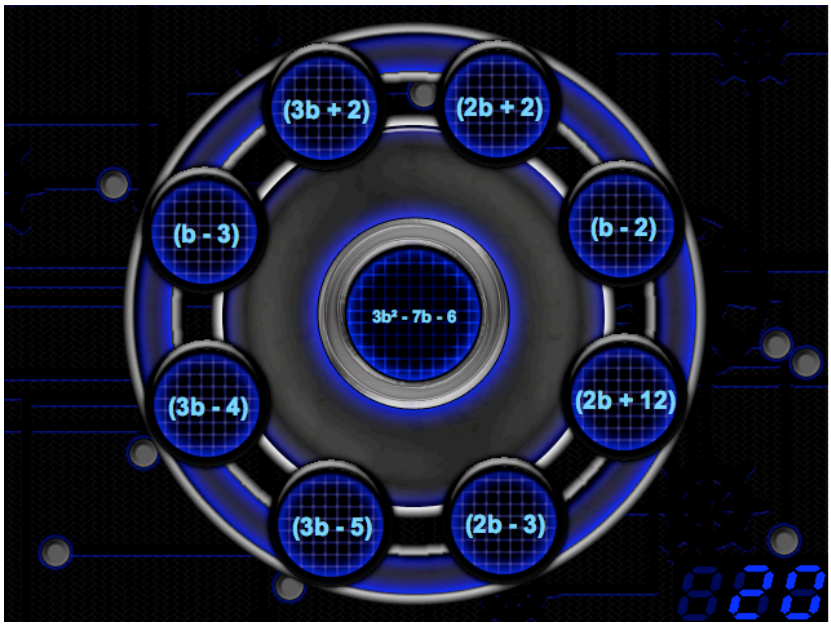


Figure 1. Match Factors players choose the factors of the polynomial in the center from a rotating ring of possibilities.

Description

Each *Match Factors* game consists of a sequence of 4 polynomials orbited by 8 possible factors. As each polynomial is displayed, players are asked to pick the matching pair of factors. If they choose correctly, the next polynomial appears. If not, they must try again before play advances.

There are three levels of play, each containing 10 games. In Level 1, polynomials have the form $x^2 + bx + c$. Level 2 polynomials have the form $ax^2 + bx + c$. Players in Level 3 must factor $ax^2 + bxy + cy^2$ polynomials.

Match Factors is suitable for individual or group play. It could also be used in a classroom setting with the whole group taking turns calling out the two factors of the expression.

Unit 9: Factoring

Instructor Overview
Project: A Cool Million

Student Instructions

Introduction
What would you do with one million dollars? In this project, you will have the opportunity to spend one million dollars on building a swimming pool complex. There are many factors to consider, including the price of construction and the size of the various elements of the complex. Of course, you will need to use algebraic factoring skills to solve the problems. Get ready for some fun with factoring!

Task
Working together with your group, you will design a swimming pool complex. The total budget for the project is one million dollars. By solving each problem algebraically, your group will determine the width of a concrete walkway, the dimensions of the kiddie pool, and the dimensions of a landscape design. At the end of the project, you will have some money left over and will be able to make decisions about adding extra elements to the pool.

Instructions
Solve each problem in order keeping careful notes along the way. Be sure to show your algebraic work completely, as it will be displayed on your final product. Once all of the problems are solved, you will create a display of the pool complex design.

1 First problem:

- The Adult Pool Area has a total area of 15,000 square feet. The Adult Pool Area consists of a rectangular swimming pool and a concrete walkway of even width surrounding all four sides of the pool. The swimming pool is 120 ft. by 70 ft.

  Hint: First draw a picture of the pool and the walkway surrounding it on all four sides and label the walkway of even width, *x*.

- Now label your picture with an expression for the total length, including the walkway. Then label your picture with an expression for the total width, including the walkway.

  Hint: How many sections of walkway are added to the length? What about to the width?

- Using the expressions for total length and width, find an expression for the total area of the rectangular pool and walkway.

In order to solve for *x*, you will need to set the expression for area equal to the 15,000 square feet. Now use your factoring skills to solve.

Hint: Don't forget to set your equation equal to zero before factoring. Is there a GCF?

- Now that you have found the width of the concrete walkway surrounding the pool, you should be able to sketch out a quick drawing of the Adult Pool Area with all dimensions labeled. This drawing will help you keep your measurements straight as you continue with the project.

2 Second problem:

- The Kiddie Pool Area has a total area of 5000 square feet. The Kiddie Pool Area consists of a rectangular pool and a rectangular concrete pad. The length of the rectangular pool is 30 feet greater than the width. The pool has a concrete pad along only one side of the pool. The dimensions of the pad are 20 ft. by *w* feet, where *w* is the width of the pool.

  Hint: First draw a picture of the pool and the concrete pad. Position the pad so that one side is along the width of the pool.

- Now label your picture with an expression for the total length, including the concrete pad. Label your picture with an expression for the total width.

  Hint: Use substitution to label the length of the pool in terms of *w*.

- Using the expressions for total length and width, find an expression for total area of the rectangular pool and concrete pad. In order to solve for w, you will need to set the expression for area equal to the 5000 square feet. Now use your factoring skills to solve.

  Hint: You will need to distribute the *w* and set the equation equal to zero in order to factor.

- Now that you have found the width of the pool, solve for the length. Again, sketch out a quick drawing of the Kiddie Pool Area with all dimensions labeled. This drawing will help you keep your measurements straight as you continue with the project.

3 Third Problem:

- A rectangular Landscape element is being added to the pool complex to beautify the grounds. If the length is 8 more than twice the width and the total area is 960 square feet, find the length and width of the rectangular landscape element.

Hint: Follow the same process as problems one and two. First, draw a picture and label. Then find an algebraic expression for total area. In this case, set that expression equal to 960 square feet. Then set the equation equal to zero and don't forget to factor out the GCF.

- Sketch out a quick drawing of the Landscape element with all dimensions labeled. This drawing will help you keep your measurements straight as you continue with the project.

4 Fourth problem:

- Keeping in mind the one million dollar budget, your group will now calculate the costs of each of the parts of the pool complex.

Type of surface	Cost per square foot
Pool (Adult or Kiddie)	$75
Landscaping	$4
Concrete	$3
Grass	$1

- The pool complex will consist of the Adult Pool Area, the Kiddie Pool Area, and the Landscape element. The outside dimensions of the complex are 150 ft. by 200 ft. The remaining area can be covered in grass, concrete, or landscaped. It will be helpful to use the following table to complete your calculations.

	Material	Total Square Footage	Cost per square foot	Total Cost
Adult Area	Pool		$75	
	Concrete		$3	
Kiddie Area	Pool		$75	
	Concrete		$3	
Landscape	Landscape		$4	
Remaining Area				
TOTAL				

Hint: Ensure that your total cost is within the one million dollar budget and check that your total area is correct by adding the areas of each piece. What should the areas of the pieces total?

Collaboration

Get together with another group to compare your answers to each of the four problems. Discuss what your group decided to do with the remaining area in the complex. Some groups may have chosen to go with the cheaper grass, while some may have chosen the more expensive landscaping.

- What if you were able to spend any remaining money on extra elements for your pool?
- Would that change how you finished the remaining area?
- What elements are missing from the pool complex?
- Do some Internet research to determine how your group would like to spend the remaining money. Include your extras in the final product.

Conclusions
Your final product will be a poster-sized display of the pool complex, including all algebra used to solve the four problems above. You may either neatly draw the pool complex or use design software such as Google SketchUp to create a professional computer generated product. You may want to use headings to separate your work for each of the three areas: Adult, Kiddie, and Landscaping.

Free Download: http://sketchup.google.com/

Instructor Notes

Assignment Procedures

Students should produce the following tables and results:

Problem 1
The width of the concrete walkway is 15 feet. The pool totals 8400 square feet and the concrete totals 6600 square feet.

$$(2x+120)(2x+70)=15{,}000$$
$$4x^2+380x-6600=0$$
$$4(x^2+95x-1650)=0$$
$$4(x-15)(x+110)=0$$
$$x=15,-110$$

Problem 2
The width of the pool is 50 feet. The total area of the pool is 4000 square feet and the total area of the concrete pad is 1000 square feet.

$$w(w+50)=5000$$
$$w^2+50w-5000=0$$
$$(w+100)(w-50)=0$$
$$w=50,-100$$

Problem 3
The width of the landscape element is 20 feet. The total area of the landscape element is 960 square feet.

$$w(2w+8)=960$$
$$2w^2+8w-960=0$$
$$2(w^2+4w-480)=0$$
$$2(w-20)(w+24)=0$$
$$w=20,-24$$

Problem 4
The cost for the remaining area will vary based on the material chosen.

	Material	Total Square Footage	Cost per square foot	Total Cost
Adult Area	Pool	8400	$75	$630,000
	Concrete	6600	$3	$19,800
Kiddie Area	Pool	4000	$75	$300,000
	Concrete	1000	$3	$3000
Landscape	Landscape	960	$4	$3840
Remaining Area		9040		
TOTAL		30000		$956,640

Recommendations:

- have students work in teams to encourage brainstorming and cooperative learning.
- assign a specific timeline for completion of the project that includes milestone dates.
- provide students feedback as they complete each milestone.
- ensure that each member of student groups has a specific job.

Technology Integration

This project provides abundant opportunities for technology integration, and gives students the chance to research and collaborate using online technology.

The following are other examples of free Internet resources that can be used to support this project:

http://sketchup.google.com/
Sketchup is a free graphics download that students may wish to use to draw the swimming pool complex. It may be helpful to demonstrate this or similar software to the entire class, so students can focus most of their time on the algebra.

http://www.moodle.org
An Open Source Course Management System (CMS), also known as a Learning Management System (LMS) or a Virtual Learning Environment (VLE). Moodle has become very popular among educators around the world as a tool for creating online dynamic websites for their students.

http://www.wikispaces.com/site/for/teachers or http://pbworks.com/content/edu+overview
Lets you create a secure online Wiki workspace in about 60 seconds. Encourage classroom participation with interactive Wiki pages that students can view and edit from any computer. Share class resources and completed student work with parents.

http://www.docs.google.com
Allows students to collaborate in real-time from any computer. Google Docs provides free access and storage for word processing, spreadsheets, presentations, and surveys. This is ideal for group projects.

http://why.openoffice.org/
The leading open-source office software suite for word processing, spreadsheets, presentations, graphics, databases and more. It can read and write files from other common office software packages like Microsoft Word or Excel and MacWorks. It can be downloaded and used completely free of charge for any purpose.

Rubric

Score	Content	Presentation
4	Your project appropriately answers each of the problems. Each of the three problems is solved algebraically by factoring. A neat, detailed table is included to display the breakdown of the costs involved.	Your project contains information presented in a logical and interesting sequence that is easy to follow. Your project is professional looking with graphics and attractive use of color.
3	Your project appropriately answers each of the problems. Each of the three problems is solved algebraically by factoring. Minor errors may be noted. A neat table is included to display the breakdown of the costs involved. Minor errors may be noted.	Your project contains information presented in a logical sequence that is easy to follow. Your project is neat with graphics and attractive use of color.
2	Your project attempts to answer each of the problems. Algebraic solutions are either not shown or guess & check is used, rather than algebra. Errors are noted. A table is included to display the breakdown of the costs involved but may not be complete. Errors are noted.	Your project is hard to follow because the material is presented in a manner that jumps around between unconnected topics. Your project contains low quality graphics and colors that do not add interest to the project.
1	Your project attempts to answer the three problems, but little to no work is shown. Major errors are noted. The breakdown of the costs involved is minimal or missing. Majoerrorsare noted.	Your project is difficult to understand because there is no sequence of information. Your project is missing graphics and uses little to no color.

Unit 9: Algebra – Factoring

Glossary

binomial	a sum of two monomials, such as $3x^2 + 7$
coefficient	a number that multiplies a variable
Distributive Property	states that the product of a number and a sum equals the sum of the individual products of the number and the addends: for all real numbers *a*, *b*, and *c*, $a(b + c) = ab + ac$
factor	for any number *x*, the numbers that can be evenly divided into *x* are called factors of *x*. For example, the number 20 has the factors 1, 2, 4, 5, 10, and 20.
factored form of a polynomial	a polynomial written as a product of factors, and each non-monomial factor has no common factors in its terms
factoring	the process of breaking a number down into its multiplicative factors. Every number x has at least the numbers 1 and x as factors.
greatest common factor (GCF)	the largest factor that two numbers have in common
grouping technique	a factoring technique involving finding common factors among groups of terms rather than among all of terms
monomial	a number, a variable, or a product of a number and one or more variables with whole number exponents, such as -5, x, and 8xy3
perfect square	any of the squares of the integers. Since 12 = 1, 22 = 4, 32 = 9, etc., 1, 4, and 9 are perfect squares
perfect square trinomial	a trinomial that is the product of a binomial times itself, such as r2 + 2rs + s2 (from (r + s)2), and r2 – 2rs + s2 (from (r – s)2)
polynomial	a monomial or sum of monomials, like 4x2 + 3x – 10
prime factor	a factor that has no factors but 1 and itself. For example, 2 is a prime factor of 12 because its only factors are 1 and 2, while 6 is not a prime factor of 12 because it has more factors than 1 and 6 (i.e. 2 and 3).
prime factorization	the process of breaking a number down into its prime factors
prime trinomial	a trinomial that cannot be factored using integers
quadratic equation	an equation that can be written in the form ax2 + bx + c = 0 where a ¹ 0. When written as y = ax2 + bx+ c the expression becomes a quadratic function.
root of an equation	any number that makes the equation true when the variable is equal to that number. That is, a solution of the equation.
trinomial	a three-term polynomial
Zero Product Property	states that if ab = 0, then either a = 0 or b = 0, or both a and b are 0

NROC Algebra 1--An Open Course
Unit 9
Mapped to Common Core State Standards, Mathematics

Algebra 1 | Factoring | Factoring Monomials and Polynomials | Factoring and the Distributive Property

Grade: 7 - Adopted 2010

STRAND / DOMAIN	**CC.7.EE.**	Expressions and Equations
CATEGORY / CLUSTER		Use properties of operations to generate equivalent expressions.
STANDARD	**7.EE.1.**	Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

Algebra 1 | Factoring | Factoring Monomials and Polynomials | Factoring Trinomials by Grouping 1

Grade: 9-12 - Adopted 2010

STRAND / DOMAIN	**CC.A.**	Algebra
CATEGORY / CLUSTER	**A-SSE.**	Seeing Structure in Expressions
STANDARD		Write expressions in equivalent forms to solve problems.
EXPECTATION	**A-SSE.3.**	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
GRADE EXPECTATION	**A-SSE.3.a.**	Factor a quadratic expression to reveal the zeros of the function it defines.

Algebra 1 | Factoring | Factoring Monomials and Polynomials | Factoring Trinomials by Grouping 2

Grade: 9-12 - Adopted 2010

STRAND / DOMAIN	**CC.A.**	Algebra
CATEGORY / CLUSTER	**A-SSE.**	Seeing Structure in Expressions
STANDARD		Write expressions in equivalent forms to solve problems.
EXPECTATION	**A-SSE.3.**	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

GRADE EXPECTATION	**A-SSE.3.a.**	Factor a quadratic expression to reveal the zeros of the function it defines.

Algebra 1 | Factoring | Factoring Special Products of Polynomials | Factoring Special Products

Grade: 9-12 - Adopted 2010

STRAND / DOMAIN	**CC.A.**	Algebra
CATEGORY / CLUSTER	**A-SSE.**	Seeing Structure in Expressions
STANDARD		Write expressions in equivalent forms to solve problems.
EXPECTATION	**A-SSE.3.**	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
GRADE EXPECTATION	**A-SSE.3.a.**	Factor a quadratic expression to reveal the zeros of the function it defines.

Algebra 1 | Factoring | Factoring Special Products of Polynomials | Solving Quadratic Equations by Factoring

Grade: 9-12 - Adopted 2010

STRAND / DOMAIN	**CC.A.**	Algebra
CATEGORY / CLUSTER	**A-SSE.**	Seeing Structure in Expressions
STANDARD		Write expressions in equivalent forms to solve problems.
EXPECTATION	**A-SSE.3.**	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
GRADE EXPECTATION	**A-SSE.3.a.**	Factor a quadratic expression to reveal the zeros of the function it defines.
STRAND / DOMAIN	**CC.A.**	Algebra
CATEGORY / CLUSTER	**A-REI.**	Reasoning with Equations and Inequalities
STANDARD		Understand solving equations as a process of reasoning and explain the reasoning.
EXPECTATION	**A-REI.1.**	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
STRAND / DOMAIN	**CC.A.**	Algebra

CATEGORY / CLUSTER	A-REI.	Reasoning with Equations and Inequalities
STANDARD		Solve equations and inequalities in one variable.
EXPECTATION	A-REI.4.	Solve quadratic equations in one variable.
GRADE EXPECTATION	A-REI.4.b.	Solve quadratic equations by inspection (e.g., for x^2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a plus-minus bi for real numbers a and b.
STRAND / DOMAIN	CC.F.	Functions
CATEGORY / CLUSTER	F-IF.	Interpreting Functions
STANDARD		Analyze functions using different representations.
EXPECTATION	F-IF.8.	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
GRADE EXPECTATION	F-IF.8.a.	Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
STRAND / DOMAIN	CC.F.	Functions
CATEGORY / CLUSTER	F-BF.	Building Functions
STANDARD		Build new functions from existing functions.
EXPECTATION	F-BF.4.	Find inverse functions.
GRADE EXPECTATION	F-BF.4.a.	Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) =2 x^3 for x > 0 or f(x) = (x+1)/(x-1) for x not equal to 1.