Unit 11: Rational Expressions and Equations

Unit 11: Rational Expressions and Equations

Unit Table of Contents

Unit 11

Lesson 1

Topic 1, Presentation – 3.7 minutes
Topic 1, Worked Example 1 – 4.1 minutes
Topic 1, Worked Example 2 – 6.6 minutes
Topic 1, Worked Example 3 – 3.8 minutes

Topic 2, Presentation – 4.2 minutes
Topic 2, Worked Example 1 – 3.6 minutes
Topic 2, Worked Example 2 – 4.8 minutes
Topic 2, Worked Example 3 – 4.1 minutes

Topic 3, Presentation – 4.3 minutes
Topic 3, Worked Example 1 – 3.1 minutes
Topic 3, Worked Example 2 – 2.9 minutes
Topic 3, Worked Example 3 – 4.6 minutes

Lesson 2

Topic 1, Presentation – 3.1 minutes
Topic 1, Worked Example 1 – 4.2 minutes
Topic 1, Worked Example 2 – 4.1 minutes
Topic 1, Worked Example 3 – 4.5 minutes

Topic 2, Presentation – 4.8 minutes
Topic 2, Worked Example 1 – 5.4 minutes
Topic 2, Worked Example 2 – 5.9 minutes
Topic 2, Worked Example 3 – 7.9 minutes
Topic 2, Worked Example 4 – 4.4 minutes

Unit 11: Rational Expressions and Equations

Instructor Notes

The Mathematics of Rational Expressions and Equations

In this unit, students will learn how to carry out basic mathematical operations on rational expressions—in other words, how to simplify, add, subtract, multiply, and divide fractions that contain polynomials. It sounds simple enough, but in fact, this unit is one of the more challenging parts of the course:

1. Students must be good at factoring trinomials to succeed, but they only just learned that technique.
2. Although most will have worked with fractions with different denominators since 5^{th} grade or so, these types of problems still confuse and intimidate many students.
3. Applying rational equations to work problems can sometimes be highly counter-intuitive.

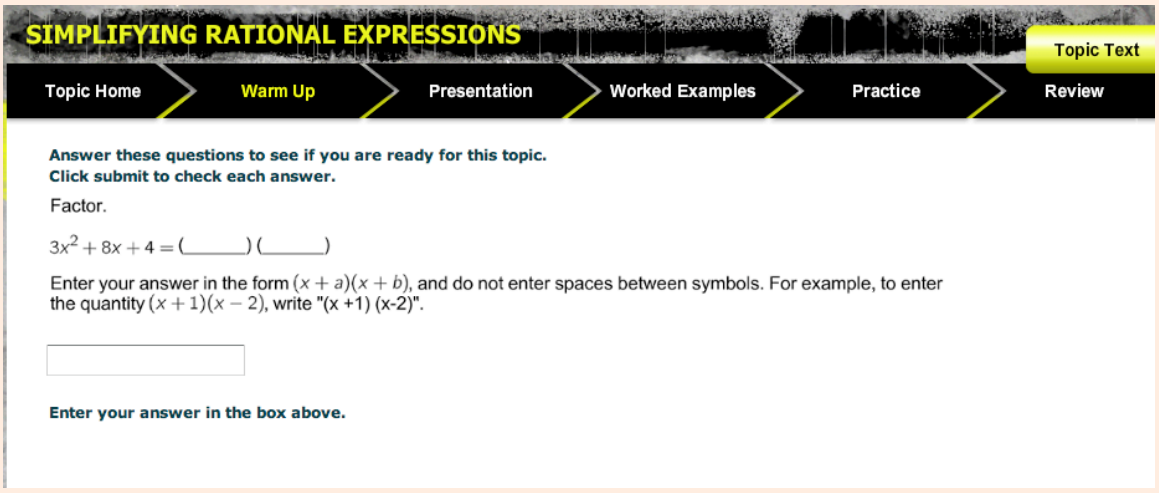
Teaching Tips: Algorithmic Challenges and Approaches

The keys to teaching students the procedures for working with rational expressions are preparation and practice.

First, students *must* have mastered the previous unit on factoring trinomials before attempting Unit 11. This work relies so heavily on that skill that it is simply impossible to learn anything meaningful about working with rational expressions without being able to factor trinomials.

The “Warm Up” to Lesson 1, Topic 1 can be used to check factoring skills:

Example



We recommend assigning the 5 Warm Up questions, and then reviewing the results carefully to see if students are ready to begin Unit 11.

It is also essential that students are comfortable finding common denominators to add and subtract fractions with unlike denominators. It's even better if they can find these common denominators by using prime factorization instead of by multiplying.

It is a good idea to review this technique by working through a few strictly numeric fractions, and then move on to applying it with polynomial fractions.

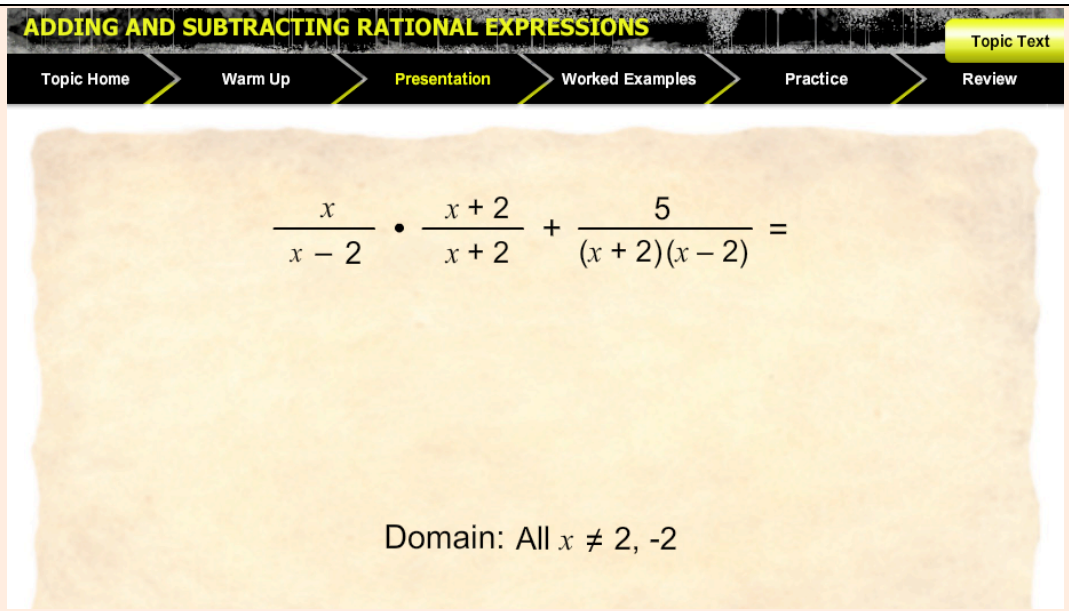
Example

Describe how to add two fractions with unlike denominators by using prime factorization:

$$\frac{3}{4} + \frac{1}{6} = \frac{3 \times 3}{4 \times 3} + \frac{1 \times 2}{6 \times 2} \text{ {multiply by } } \frac{3}{3} \text{ and } \frac{2}{2}\text{}$$

$$= \frac{9}{12} + \frac{2}{12} = \frac{11}{12} \text{ {add the numerators}}$$

Then show how to generalize this method when working with rational expressions. The presentation in Topic 1 explains this clearly:



Notice here how the fraction on the left is being multiplied by $\frac{x+2}{x+2}$ i.e. 1, in order to make the denominators equal so the numerators can be added. This explanation will be very difficult for students to follow if they are not familiar with performing the same operation with numeric fractions.

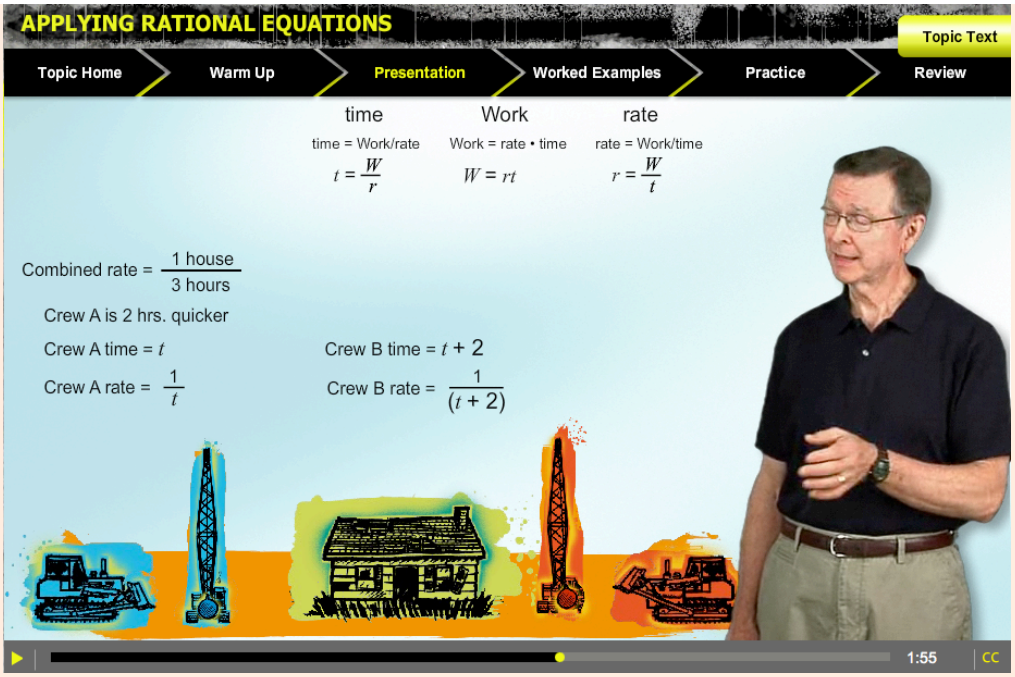
Students can review factorization and run through practice problems with feedback here:

- http://www.math.com/school/subject1/lessons/S1U4L3GL.html

Teaching Tips: Conceptual Challenges and Approaches

The real conceptual challenge in unit 11 is the logic behind the use of rational equations to solve work problems. It's important to very carefully explain the idea that individual rates of work can be added together to get a combined rate. Be sure to work through several of these problems as a group, using real world examples that students can easily understand.

Example
The video for Lesson 2, Topic 2 discusses an example where two wrecking crews are demolishing a house:



You may find it beneficial to watch this particular video with the whole class and ask key questions to ensure students understand all of the critical pieces in this puzzle.

The scenario is that if both wrecking crews work together, they can level 1 house in 3 hours, and if they worked individually, crew B would take 2 hours longer than crew A. At this stage it would be worthwhile to ask the class how long crew A would take to level the house, and how long crew B would take.

Do not be surprised if at this point the students make suggestions like “crew A would take 1 hour, and crew B would take 2 hours.” Students will intuitively want to add their individual times together to make 3, the amount of time the crews take to do the job working together. Students need to be allowed to make this mistake, and be given time to discuss why this is impossible–it’s not hard to realize that a crew working alone will actually take longer than two crews working together.

This is probably one of the first times that students have encountered this type of problem in a mathematics class and they will need some time to discuss it and build their intuitive understanding of what is going on.

Once students understand how rates can be combined in a rational expression, they'll have a much easier time writing and solving these kinds of word problems. They can use this knowledge as a quick check on their answers—does a calculated rate make sense in comparison to the given rates?

Summary
Unit 11 tackles some fairly complex procedures and difficult ideas in working with rational expressions and equations. We suggest you make sure students are fluent with factoring and working with numerical fractions before you begin. Then as they work with polynomial fractions, and especially rate-related word problems, give them time to make

Unit 11: Rational Expressions and Equations

Instructor Overview
Tutor Simulation: Conserving Water

Purpose

This simulation is designed to challenge a student's understanding of rational expressions and equations. Students will be asked to apply what they have learned to solve a real world problem by demonstrating understanding of the following areas:

- Rational Expressions
- Rational Equations
- Solving Rational Equations
- Solving Work Problems
- Quadratic Equations
- Factoring

Problem

Students are given the following problem:

> Your school is trying to conserve water, including the water used to irrigate the football field. A student at your school, Yun, is analyzing the football field's two sprinkler systems.
>
> He has some of the measurements he needs for the project, but could use some help with the calculations.

Recommendations

Tutor simulations are designed to give students a chance to assess their understanding of unit material in a personal, risk-free situation. Before directing students to the simulation,

- make sure they have completed all other unit material.
- explain the mechanics of tutor simulations
  - Students will be given a problem and then guided through its solution by a video tutor;
  - After each answer is chosen, students should wait for tutor feedback before continuing;
  - After the simulation is completed, students will be given an assessment of their efforts. If areas of concern are found, the students should review unit materials or seek help from their instructor.
- emphasize that this is an exploration, not an exam.

Unit 11: Rational Expressions and Equations

Instructor Overview
Puzzle: We Can Work It Out

Objective

We Can Work It Out is a puzzle that requires students to write and solve rational expressions in order to find the answers to rate problems. It tests a player's grasp of the reasoning behind using polynomial fractions to calculate combined rates, as well as their ability to add and subtract these types of expressions.

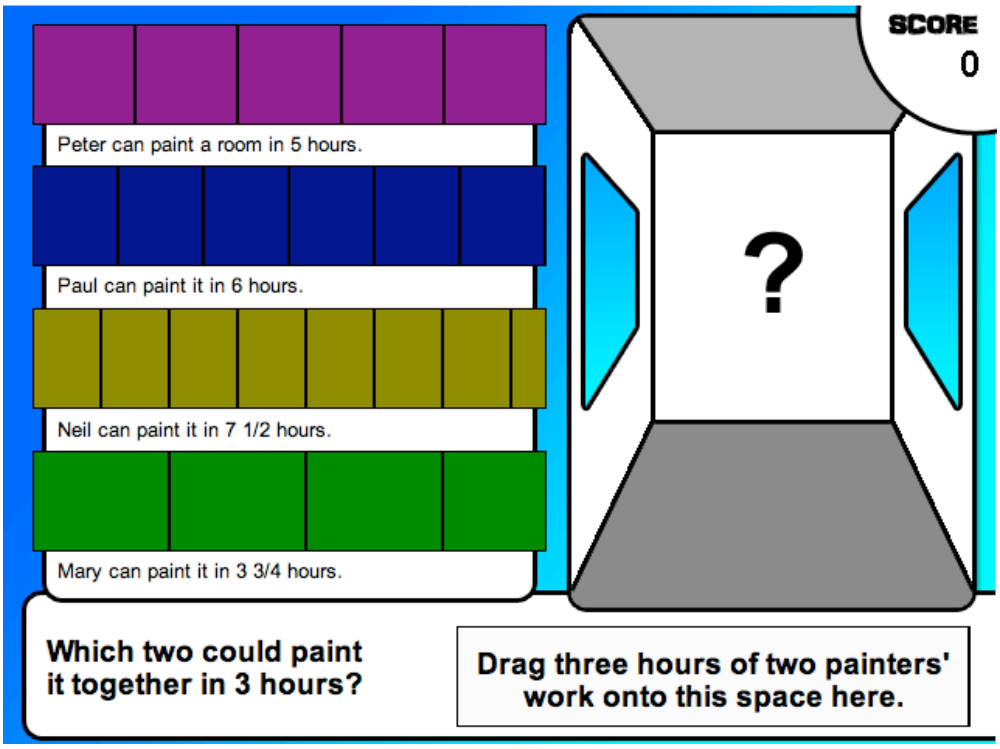


Figure 1. *Players need to use the rate = work/time formula to write a rational equation that will lead to the correct combination of workers.*

Description

This puzzle presents 10 workplace scenarios each that ask players to chose the combination of workers that will complete a given task in a specified amount of time. To do so, players must write a rational expression to describe the output of each of several workers. They then must calculate how to add the rates to find a set of workers that together will produce the desired group rate.

Players earn points for choosing the right combination of workers. The puzzle does not advance until the correct answer is reached.

We Can Work It Out is most effective as a single player game, so that each player has time to work through the problems at his own speed. It could be used in a classroom by asking the group to shout out or take turns calculating rates and suggesting combinations.

Unit 11: Rational Expressions and Equations

Instructor Overview
Project: Work It Out!

Student Instructions

Introduction
Solving work problems in algebra requires a solid understanding of rational equations. By now you should be comfortable with a classic problem such as:

> *If it takes Bob 3 hours to paint a room and it takes Jeff 5 hours to paint the same room, how many hours would it take if the two painters work together?*

Many students will attempt to take an average and come up with an answer of 4 hours, but 4 hours doesn't make sense. Why would it take longer than Bob working alone? Surely the two painters would complete the room in less than 3 hours. Using the algebra work model, we discover that the room should be painted in slightly less than two hours. The work model allows us to consider each painter's contribution as a rate and add the two rates together to calculate a time projection for the pair painting the room together.

Task
Working with your group, you will first solve a work problem algebraically. Once you are comfortable with solving the problem, you will design a task for your fellow students to perform in class. First you will time three students completing the task individually and then use the work model to determine how long it should take for the three students to complete the task while working together. Of course models are not always perfect, so your next step will be to see how close your model comes to reality. Get ready for some hard work!

Instructions
Complete each problem in order keeping careful notes along the way. At the conclusion, you will design a website to showcase your project. You may want to use a camera to record video or take pictures as you are working.

1 First problem: Solve a problem

- Students A, B, and C can complete a puzzle in 20 minutes, 15 minutes, and 30 minutes, respectively. If all three students work together, how long should it take to complete the puzzle?

- First, find the rate for Student A. Then, find the rate for Student B. Finally, find the rate for Student C.

Hint: Remember that $r = \frac{W}{t}$. The work being completed is one puzzle.

- If the students work together, their rates will need to be added together. What is the total rate when all three students work together?

  Hint: Don't forget to get common denominators. What is the LCM of 15, 20, and 30?

- Now substitute the total rate into the work model, $r = \frac{W}{t}$. The variable *W* is still one because the amount of work to be completed is one puzzle. Then, solve for *t.*

  Hint: Express your answer in terms of minutes and seconds.

2 Second problem: Collect individual data

1. Now that you have successfully used the work model to solve a real-life problem, get with your group to discuss what type of activity could be timed within your classroom. The students from the first problem completed a puzzle. Determine what type of task your group would like to time people performing. The task needs to take at least 2 minutes but no more that 10 minutes for a student to perform alone.

   Some ideas include: shooting 50 free throws, passing out 25 papers, solving a set of math problems, building a card tower, building a figure from blocks.

2. Choose a task that will highlight your group's creativity and interests. You may need to bring in materials from home or ask to borrow materials from your teacher. Be sure to get approval from your teacher before continuing.

3. Once your idea is approved, find three willing participants to try the task. Time each student individually and record the time. For simplicity, you will want to round each student's time to the nearest ½ of a minute.

   Hint: In order to have accurate data, ensure that each student has not seen the task being performed before being timed. Don't forget to capture video or photos during the tasks.

3 Third problem: Calculate using the work model

- Now that you have each student's time individually, use the work model and your knowledge of rational equations to determine how

long it should take the three students to complete the task if they work together.

Hint: First find the rate for each student. Then add the rates of all three together, being careful to get common denominators. Once you have the total rate, substitute the rate into the work model: $r = \frac{W}{t}$. Remember that *W* will be one. Then solve for *t*.

- The solution is the amount of time it should take if all three students work together to complete the task. Do you think that the time is accurate? Why or why not?

4 Fourth problem: Collect group data and compare

1. Gather the same three students that were timed before and ask them to work together to complete the task. Time how long it takes for the task to be completed when the three students work together.

2. Compare the actual time to the projected time by calculating the percent error.

   Hint: Percent error is calculated by first finding the change between the actual time and the projected time. The absolute value of this change is then divided by the projected or theoretical time. The formula for percent error is as follows:

$$\frac{\text{Projected} - \text{Actual}}{\text{Projected}} \bullet 100$$

5 Fifth Problem: Discussion

3. Was the actual time to complete the task more or less than the projected time? What factors might have contributed to that result?

4. What steps might your group have taken in order to minimize the percent error of your experiment?

5. When completing your task, did you allow three people to work together effectively? Why or why not?

Collaboration:
Work together with your group to write your own work model word problem. If needed use the examples from the lesson on Rational Equations to help guide your work. On a separate piece of paper, solve the problem within your group. Trade only problems, not solutions, with a neighboring group. Work together within your group to find the solution to the new word problem that you got from the other group. When your group agrees on a solution, check your solution against the original solution from the group that authored

the problem. Do the answers agree? If not, compare the solutions and work together to determine which solution is correct and why.

Conclusions
Work together within your group to create a website to highlight your project. Websites can be created for free at www.weebly.com. Consider importing pictures or videos of the students completing your task to add a touch of creativity and interest. The website should contain solutions to each of the five problems.

Instructor Notes

Assignment Procedures

Students should produce the following tables and results:

Problem 1
Algebra students often have difficulty when the variable to be solved for is in the denominator. Students work extensively with proportions in earlier grades. When the students approach the problem as a proportion and then cross-multiply, the variable is easily manipulated from the denominator.

Problem 2
The task needs to be something that gives three people equal chances to contribute. Some ideas include:

- Typing a familiar tongue twister 100 times – each student can work on his own computer to contribute to the total.
- Shooting 50 free throws – position each student at his own basketball hoop so that there is no interference.

Once students pick a task, have them write out their goal and the procedure for collecting data. This written proposal can then be kept in order to ensure that each group is indeed performing only approved experiments. Student safety needs to be maintained and this step is important when students are designing their own experiments.

Recommendations:

- assign a specific timeline for completion of the project that includes milestone dates.
- provide students feedback as they complete each milestone.
- ensure that each member of student groups has a specific job.

Technology Integration

This project provides abundant opportunities for technology integration, and gives students the chance to research and collaborate using online technology.

The student instructions suggest using www.weebly.com to set up websites to chronicle the projects.

The following are other examples of free Internet resources that can be used to support this project:

http://www.moodle.org
An Open Source Course Management System (CMS), also known as a Learning Management System (LMS) or a Virtual Learning Environment (VLE). Moodle has become very popular among educators around the world as a tool for creating online dynamic websites for their students.

http://www.wikispaces.com/site/for/teachers or http://pbworks.com/content/edu+overview allows you to create a secure online Wiki workspace in about 60 seconds. Encourage classroom participation with interactive Wiki pages that students can view and edit from any computer. Share class resources and completed student work with parents.

http://www.docs.google.com
Allows students to collaborate in real-time from any computer. Google Docs provides free access and storage for word processing, spreadsheets, presentations, and surveys. This is ideal for group projects.

http://why.openoffice.org/
The leading open-source office software suite for word processing, spreadsheets, presentations, graphics, databases and more. It can read and write files from other common office software packages like Microsoft Word or Excel and MacWorks. It can be downloaded and used completely free of charge for any purpose.

Rubric

Score	Content	Presentation
4	Your project appropriately answers each of the problems. Completed data tables and step-by-step algebraic solutions are included. Evidence of careful data collection is apparent. The chosen task effectively allows three students to work together to complete the task.	Your project contains information presented in a logical and interesting sequence that is easy to follow. Your project is professional looking with graphics and attractive use of color.
3	Your project appropriately answers each of the problems. Completed data tables and algebraic solutions are included. All steps may not be shown. Evidence of careful data collection is apparent. The chosen task effectively allows three students to work together to complete the task. Minor errors may be noted.	Your project contains information presented in a logical sequence that is easy to follow. Your project is neat with graphics and attractive use of color.
2	Your project partially answers each of the problems. Partially completed data tables and algebraic solutions are included. All steps may not be	Your project is hard to follow because the material is presented in a manner that jumps around between unconnected topics.

	shown. Evidence of data collection is apparent although errors are present. The chosen task allows three students to work together to complete the task, but is not ideal. Errors may be noted.	Your project contains low quality graphics and colors that do not add interest to the project.
1	Your project attempts to answer some of the problems. Partially completed data tables and algebraic solutions are included. Very little work is shown. Evidence of data collection is apparent although errors are present. The chosen task allows three students to work together to complete the task, but is not ideal. Major errors are noted.	Your project is difficult to understand because there is no sequence of information. Your project is missing graphics and uses little to no color.

Unit 11: Rational Expressions and Equations

domain	the set of all possible inputs of a function which allow the function to work
excluded value	a value for a variable that is not allowed in an expression, such as a variable in a rational expression that would make the denominator equal zero
extraneous solution	a solution that results from solving an equation that is not a valid solution in the original equation
greatest common factor	the largest number or expression that will divide a number or expression exactly
least common denominator	the smallest number or expression that is a multiple of all the denominators in a group of fractions or rational expressions
least common multiple	the smallest number or expression that is a multiple of a group of numbers or expressions
prime factorization	the process of breaking a number down into its prime factors
rational equation	an equation that contains one or more rational expressions
rational expression	a fraction with a polynomial in the numerator and/or denominator

NROC Algebra 1--An Open Course
Unit 11: Rational Expressions and Equations
Mapped to Common Core State Standards, Mathematics

Unit 11, Lesson 1, Topic 1: Simplifying Rational Expressions

Grade: **9-12** - Adopted **2010**

STRAND / DOMAIN	**CC.A.**	Algebra
CATEGORY / CLUSTER	**A-SSE.**	Seeing Structure in Expressions
STANDARD		Interpret the structure of expressions.
EXPECTATION	**A-SSE.2.**	Use the structure of an expression to identify ways to rewrite it. For example, see x^4 - y^4 as (x^2)^2 - (y^2)^2, thus recognizing it as a difference of squares that can be factored as (x^2 - y^2)(x^2 + y^2).
STRAND / DOMAIN	**CC.A.**	Algebra
CATEGORY / CLUSTER	**A-APR.**	Arithmetic with Polynomials and Rational Functions
STANDARD		Rewrite rational expressions.
EXPECTATION	**A-APR.6.**	Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.
STRAND / DOMAIN	**CC.F.**	Functions
CATEGORY / CLUSTER	**F-IF.**	Interpreting Functions
STANDARD		Interpret functions that arise in applications in terms of the context.
EXPECTATION	**F-IF.5.**	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

Unit 11, Lesson 1, Topic 2: Multiplying and Dividing Rational Expressions

Grade: **9-12** - Adopted **2010**

STRAND / DOMAIN	**CC.A.**	Algebra
CATEGORY / CLUSTER	**A-SSE.**	Seeing Structure in Expressions
STANDARD		Interpret the structure of expressions.

EXPECTATION	**A-SSE.2.**	Use the structure of an expression to identify ways to rewrite it. For example, see x^4 - y^4 as (x^2)^2 - (y^2)^2, thus recognizing it as a difference of squares that can be factored as (x^2 - y^2)(x^2 + y^2).
STRAND / DOMAIN	**CC.A.**	Algebra
CATEGORY / CLUSTER	**A-APR.**	Arithmetic with Polynomials and Rational Functions
STANDARD		Rewrite rational expressions.
EXPECTATION	**A-APR.6.**	Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.
EXPECTATION	**A-APR.7.**	(+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Unit 11, Lesson 1, Topic 3: Adding and Subtracting Rational Expressions

Grade: **9-12** - Adopted **2010**

STRAND / DOMAIN	**CC.A.**	Algebra
CATEGORY / CLUSTER	**A-SSE.**	Seeing Structure in Expressions
STANDARD		Interpret the structure of expressions.
EXPECTATION	**A-SSE.2.**	Use the structure of an expression to identify ways to rewrite it. For example, see x^4 - y^4 as (x^2)^2 - (y^2)^2, thus recognizing it as a difference of squares that can be factored as (x^2 - y^2)(x^2 + y^2).
STRAND / DOMAIN	**CC.A.**	Algebra
CATEGORY / CLUSTER	**A-APR.**	Arithmetic with Polynomials and Rational Functions
STANDARD		Rewrite rational expressions.
EXPECTATION	**A-APR.6.**	Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.
EXPECTATION	**A-APR.7.**	(+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Unit 11, Lesson 2, Topic 1: Solving Rational Equations

Grade: **7** - Adopted **2010**

STRAND / DOMAIN	**CC.7.MP.**	Mathematical Practices
CATEGORY / CLUSTER	**7.MP.6.**	Attend to precision.

Grade: **8** - Adopted **2010**

STRAND / DOMAIN	**CC.8.MP.**	Mathematical Practices
CATEGORY / CLUSTER	**8.MP.6.**	Attend to precision.

Grade: **9-12** - Adopted **2010**

STRAND / DOMAIN	**CC.MP.**	Mathematical Practices
CATEGORY / CLUSTER	**MP-6.**	Attend to precision.
STRAND / DOMAIN	**CC.A.**	Algebra
CATEGORY / CLUSTER	**A-REI.**	Reasoning with Equations and Inequalities
STANDARD		Understand solving equations as a process of reasoning and explain the reasoning.
EXPECTATION	**A-REI.1.**	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
EXPECTATION	**A-REI.2.**	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
STRAND / DOMAIN	**CC.F.**	Functions
CATEGORY / CLUSTER	**F-BF.**	Building Functions
STANDARD		Build new functions from existing functions.
EXPECTATION	**F-BF.4.**	Find inverse functions.
GRADE EXPECTATION	**F-BF.4.a.**	Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) =2 x^3 for x > 0 or f(x) = (x+1)/(x-1) for x not equal to 1.

Unit 11, Lesson 2, Topic 2: Applying Rational Equations

Grade: **7** - Adopted **2010**

STRAND / DOMAIN	**CC.7.MP.**	Mathematical Practices
CATEGORY / CLUSTER	**7.MP.6.**	Attend to precision.
STRAND / DOMAIN	**CC.7.EE.**	Expressions and Equations
CATEGORY / CLUSTER		Use properties of operations to generate equivalent expressions.
STANDARD	**7.EE.2.**	Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."

Grade: **8** - Adopted **2010**

STRAND / DOMAIN	**CC.8.MP.**	Mathematical Practices
CATEGORY / CLUSTER	**8.MP.6.**	Attend to precision.

Grade: **9-12** - Adopted **2010**

STRAND / DOMAIN	**CC.MP.**	Mathematical Practices
CATEGORY / CLUSTER	**MP-6.**	Attend to precision.
STRAND / DOMAIN	**CC.A.**	Algebra

CATEGORY / CLUSTER	**A-REI.**	Reasoning with Equations and Inequalities
STANDARD		Understand solving equations as a process of reasoning and explain the reasoning.
EXPECTATION	**A-REI.1.**	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
EXPECTATION	**A-REI.2.**	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
STRAND / DOMAIN	**CC.F.**	Functions
CATEGORY / CLUSTER	**F-BF.**	Building Functions
STANDARD		Build new functions from existing functions.
EXPECTATION	**F-BF.4.**	Find inverse functions.
GRADE EXPECTATION	**F-BF.4.a.**	Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) =2 x^3 for x > 0 or f(x) = (x+1)/(x-1) for x not equal to 1.