Unit 10: Quadratic Functions

Unit 10: Quadratic Functions

Learning Objectives

Lesson 1: Quadratic Functions

Topic 1: Graphing Quadratic Functions

Learning Objectives

- Graph quadratic equations on the coordinate plane;
- Define and identify the roots of a quadratic equation.

Topic 2: Solving Quadratic Equations by Completing the Square

Learning Objectives

- Solve quadratic equations by completing the square.

Topic 3: Solving Quadratic Equations Using the Quadratic Formula

Learning Objectives

- Solve quadratic equations using the quadratic formula.

Lesson 2: Applying Quadratic Functions

Topic 1: Applications of Quadratic Functions

Learning Objectives

- Apply quadratic functions to real world situations in order to solve problems.

Topic 2: Systems of Non-Linear Equations

Learning Objectives

- Solve systems of equations involving linear, quadratic, and other non-linear functions.
- Analyze the domain and range of non-linear functions.

Unit 10: Quadratic Functions

Instructor Notes

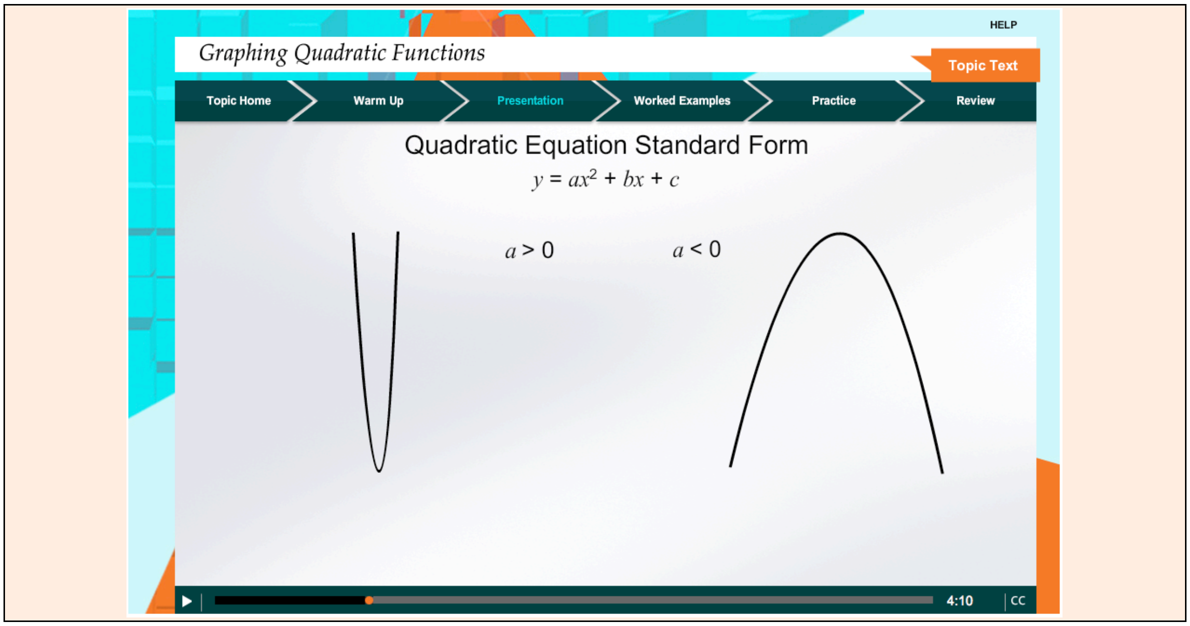
The Mathematics of Quadratic Functions

The new key concept in this unit is the graph of the quadratic function. Students will learn not just how to graph these functions, but also how to predict the shape, location, and direction of a parabola from its equation. By the end of the unit, they'll also know how to solve quadratic equations by completing the square and by using the quadratic formula. Students will also have experience applying these techniques to systems of non-linear equations. This unit includes a topic on applications of quadratic functions, so the students can now start to put the procedures and formulas learned in the previous few units to good use, and solve real-world math problems that can only be tackled using these new and powerful techniques.

Teaching Tips: Conceptual Challenges and Approaches

At this point in the course, students are comfortable with the important features of linear functions—the slope and x- and y-intercepts of a straight line. But now they'll need to understand a larger set of attributes to work with quadratic functions—the roots, y-intercept, vertex, axis of symmetry, width and direction of a parabola. This extra complexity can be challenging.
The key is to help students see the connections between the equation and the graph. Make sure that the students understand which parts of the equation control the various characteristics of the graph.

Example
The presentation for Lesson 1, Topic 1 discusses the connection between the coefficients of the quadratic equation in standard form and the shape (and direction) of the resulting parabola.



Hands-on Opportunities

Hands-on work is also essential for students to fully grasp the equation-parabola relationship. Make sure students get a lot of practice drawing graphs. The Topic Text of this unit includes 2 manipulatives that enable students to play with the coefficients of a quadratic equation.

- Axis of Symmetry (Lesson1, Topic 1)

  This manipulative let's student explore the movement of the vertex and axis of symmetry of a parabola as the coefficients of an equation are changed.

- Graphing a Parabola (Lesson 1, Topic 2)

  This manipulative lets students change the values of a, b, and c in the equation $y = ax^2 + bx + c$, and see how that affects the shape and direction of the resultant parabola.

Students can learn quite a bit playing with these manipulatives on their own. In a group setting, you can project them on a screen or an interactive whiteboard to model and discuss how the coefficients control the parabola.

There are also applets on the web that model parabolas:

- http://nlvm.usu.edu/en/nav/frames_asid_109_g_4_t_2.html. This can be used to explore the parabola of a quadratic equation in factored form.

Teaching Tips: Algorithmic Challenges and Approaches

One of the most difficult procedures in this unit is "Completing the Square." This procedure is complex, and it is also unnecessary in the context of Algebra 1 because the quadratic formula can be used to solve all quadratic equations. We include it in this unit because it does help students understand quadratic equations more fully, and also because the technique is essential in later mathematics courses such as Algebra 2 and Pre-Calculus.

It is very helpful to use visual models to explain the reasoning behind completing the square.

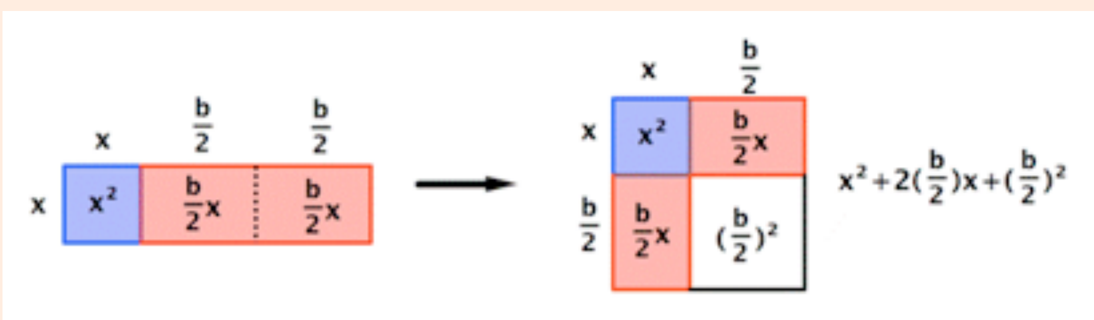
Example

Here's a problem from the text of Lesson 1, Topic 2:

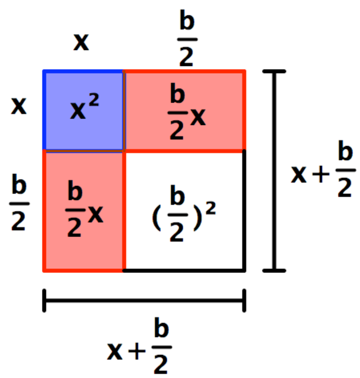
"Completing the Square" does exactly what it says—it takes something that probably is not a square and makes it one. We can illustrate this idea using an area model of the binomial $x^2 + bx$*:*

In this example, the area of the overall rectangle is given by $x(x + b)$*.*

Now let's make this rectangle into a square. First, we'll divide the red rectangle with area bx *into two equal rectangles each with area* $\frac{b}{2}x$*. Then we'll rotate and reposition one of them. We haven't changed the size of the red area—it still adds up to* bx*.*



Here comes the cool part—do you see that when the white square is added to the blue and red regions, the whole shape is now a square too? In other words, we've "completed the square!" By adding the quantity $\left(\frac{b}{2}\right)^2$ *to the original binomial, we've made a square, a square with sides of* $x+\frac{b}{2}$*:*



$$x^2+bx+\left(\frac{b}{2}\right)^2=x^2+2\left(\frac{b}{2}\right)x+\left(\frac{b}{2}\right)^2=\left(x+\frac{b}{2}\right)^2$$

After seeing the process modeled, the phrase "Completing the Square" will have meaning as students are literally finding a number $\left(\frac{b}{2}\right)^2$ that will allow the terms to be rearranged into a square.

Hands-on Opportunities

For additional practice, students can use the virtual manipulative found here [MAC users will need to copy/paste the url into a browser]:

- http://courses.wccnet.edu/~rwhatcher/VAT/CompletingTheSquare/

Students should produce both visual and algebraic solutions as they work with this tool. Gradually discontinue the use of the visual tools as students develop symbolic fluency with this procedure.

Students can also see step-by-step examples of different takes on the visual and algebraic manipulations involved in completing the square at these sites:

- http://www.regentsprep.org/Regents/math/algtrig/ATE12/completesqlesson.htm
- http://www.mathsisfun.com/algebra/completing-square.html

Summary

This unit teaches students to graph, manipulate, and solve quadratic functions. There are a number of new ideas introduced, like all the parts and determinants of parabolas. There are also some difficult techniques to be learned—completing the square and applying the quadratic formula.

Although challenging, these concepts will allow students to pull together work from earlier units to solve real-world math problems. The way these new skills and procedures are developed in this course will also form a solid foundation for students as they move into higher, more advanced mathematics courses.

We recommend teaching this material by beginning with a thorough grounding in the connections between each part of the quadratic equation and the characteristics of the parabola it describes. As each new idea or procedure is introduced, use visual models and manipulatives to help students make sense of the algebra.

Unit 10: Quadratic Functions

Instructor Overview
Tutor Simulation: Rocket Trajectory

Purpose

This simulation is designed to challenge a student's understanding of quadratic equations. Students will be asked to apply what they have learned to solve a real world problem by demonstrating understanding of the following areas:

- Visualizing Problems
- Drawing Diagrams from Word Problems
- Trajectories
- Parabolas
- Quadratic Equations
- Quadratic Formula

Problem

Students are given the following problem:

> Xavier needs help with his science experiment. He built a rocket with a camera attached, but needs help calculating the trajectory so he can set the camera timers. It's up to you to help him work out the timing—and a few other details—with trajectory calculations. Be sure you have pencil and paper handy ... you'll be needing it for the math and for a diagram that will help you visualize the problem.

Recommendations

Tutor simulations are designed to give students a chance to assess their understanding of unit material in a personal, risk-free situation. Before directing students to the simulation,

- make sure they have completed all other unit material.
- explain the mechanics of tutor simulations
    - Students will be given a problem and then guided through its solution by a video tutor;
    - After each answer is chosen, students should wait for tutor feedback before continuing;
    - After the simulation is completed, students will be given an assessment of their efforts. If areas of concern are found, the students should review unit materials or seek help from their instructor.
- emphasize that this is an exploration, not an exam.

Unit 10: Quadratic Functions

Instructor Overview
Puzzle: Shape Shifter

Objective

Shape Shifter is a manipulative puzzle that tests a student's understanding of the graphs of quadratic functions. Players are given the vertex form of the equation for a parabola, $y = a(x – h)^2$, which describes the shape, direction, and position of the parabola on a graph. They're then asked to tweak the coefficients of the equation until its parabola achieves a desired shape.

This game reinforces how the values in an equation control the appearance and placement of its graph. Because each parabola is manipulated to match the curve of a real world-world object—bridge cables, the support legs of the Eiffel Tower, the path of water in a fountain, a satellite dish, and even the familiar double curve of McDonald's golden arches—players also gain an appreciation for the prevalence of parabolas and the importance of quadratic functions in their everyday lives.

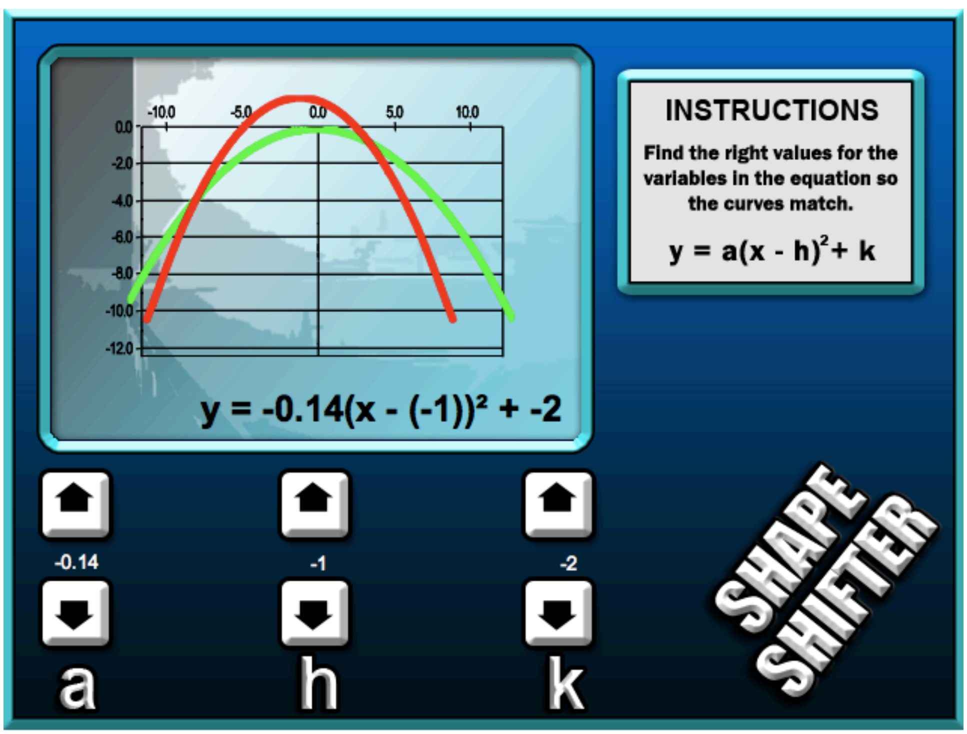


Figure 1. Shape Shifter asks players to change the coefficients in a quadratic function so that its parabola matches a given shape.

Description

Shape Shifter takes players through a sequence of ten puzzles. Each puzzle shows a quadratic equation in vertex form and two parabolas. One parabola, shown in red, is the graph of the given equation. Players are asked to use buttons to change the coefficients of the equation until its graph matches the second parabola, shown in green. When they've done so, the parabolas are replaced by a picture of the real-world object they model. Players can then move on to the next puzzle.

Shape Shifter is designed as a single player game, but could be used in a classroom by asking the group to shout out or take turns suggesting which variables need to be adjusted in which direction for the curve to fall into place.

Unit 10: Quadratic Functions

Instructor Overview
Project: Ready, Aim, Fire!

Student Instructions

Introduction
Mathematical models are used to help make real-life decisions. Models are developed by analyzing data patterns and finding the equation that best fits the data. Many models are based on a linear pattern where the dependent variable increases or decreases at a fixed rate based on the independent variable. Models of motion are often more complex quadratic models based on a variety of factors including the pull of gravity.

Task
Working together with your group, you will collect and analyze experimental data. Once your data is collected and recorded, you will use your graphing calculator to develop models based on the data. Using your equations, you will then get to test the accuracy of the models. Have you ever wanted to shoot rubber bands at school? Now is your chance!

Instructions
Materials Needed: rubber bands, ruler, protractor, measuring tape, and lab safety goggles; TI-83+ or 84+ graphing calculator to calculate the regression equations; digital camera to capture the experiment; and, a small action figure for the conclusion of the experiment.

Complete each problem in order keeping careful notes of the results. Also, be sure to take lots of pictures of the experiment. You will create a multimedia presentation of your results at the conclusion of the project.

1. First problem: Horizontal Flight Distance vs. Stretch of Rubber Band
   - This experiment is best performed outdoors on an open, level surface. Gather all of the necessary materials, including paper and pencil to record the results, and head outside. Within your group, assign the following tasks: spotter, recorder, holder, and launcher. In order to maintain safety, all group members should wear lab safety goggles to protect themselves from flying rubber bands.

   - The holder will hold the ruler level at about waist height. The launcher will place one end of the rubber band on the end of the ruler and pull back the elastic to measure the starting length at rest. (At this point the rubber band should not be stretched. This measurement is just the starting point.) For the first trial, the launcher will stretch the rubber band 1 cm beyond the starting point and release. The spotter will measure the horizontal flight distance and the recorder will record the results in the table below.

Hint: For more accurate data, the spotter needs to take note of where the rubber band first hits the ground, and measure to that point, rather than where the rubber band finally comes to rest.

- Repeat the 1 cm stretch for a total of three trials, then, find the average of the three trials and record. Continue launching rubber bands at 2 cm, 3 cm, 4 cm, etc. until the table is completed or the rubber band breaks.

Stretch beyond rest (in cm)	Trial 1	Trial 2	Trial 3	Average horizontal distance traveled (in cm)
1 cm				
2 cm				
3 cm				
4 cm				
5 cm				
6 cm				
7 cm				
8 cm				
9 cm				
10 cm				

2 Second problem: Use your TI-83+ or 84+ graphing calculator to create a scatter plot of your data.

NOTE: Clear the STAT area of your calculator by pushing the STAT key, select 4:ClrList, 2nd ,L1, enter a comma from the keyboard, 2nd, L2, and then ENTER. The line should look like this prior to pressing ENTER:

ClrList L_1,L_2

- Enter your data into a table by pushing the STAT key and then ENTER. Enter the stretch beyond rest value into L1 and the average distance value into L2. Be sure that you have 10 entries in the L1 column and 10 entries in the L2 column before going on to the next step. (Hint: If you have made an error, use the arrow keys to highlight the error and then simply type over the error.)

- Once your data is in the table, create a scatter plot by pushing 2nd and then Y=. Hit ENTER and then move the cursor, using the arrows, to ON and hit ENTER. Now to see the points on the graph, push ZOOM 9. Each data point will now appear on your screen.

- Now we will use the calculator to find the line of best fit of the data. Push the STAT key then use the right arrow to highlight CALC. Push the number 4, and then ENTER. The calculator will give the line of best fit in the form *ax*+*b*, where *a* is the slope and *b* is the y-intercept. Record the given equation on your paper with each coefficient to the nearest thousandth.

- To see how closely the line fits with the data, push Y= and enter the equation into the calculator. Then push the GRAPH key. You should now see the data points with the line of best fit.

- Record various observations about the data points and the line of best fit. How close are the points to the line? Are there any points that are very far away? What might have caused the discrepancy?

- You will need to include a graph of your data in your final project. You can either neatly graph the data and line of best fit by hand and take a picture or use a computer generated graphing program, such as GeoGebra. The GeoGebra program can be downloaded for free at http://www.geogebra.org/cms/en/download.

3 Third problem: Horizontal Flight Distance vs. Angle of elevation

- This experiment will require the holder and the launcher to work together carefully. Again, the holder will keep the ruler level at about waist height. The launcher will place one end of the rubber band on the end of the ruler and pull back the elastic to measure the starting length at rest. For all the trials, the launcher will stretch the rubber band 5 cm beyond the starting point and release. Just like in the previous experiment, the spotter will measure the horizontal flight distance and the recorder will record the results in the table below.

- The difference is that now the ruler will be positioned with varying angles of elevation. The first trials, where the ruler is level, is an angle of 0 degrees elevation. To achieve an angle of 10 degrees, the ruler will need to be positioned on a protractor and angled up to the 10-degree mark. Remember, the rubber band should be stretched 5 cm and three trials should be done at each angle of elevation.

Angle of elevation (in degrees)	Trial 1	Trial 2	Trial 3	Average horizontal distance traveled (in cm)
0				
10				
20				
30				
40				
50				
60				
70				
80				
90				

4 Fourth problem: Use your TI-83+ or 84+ graphing calculator to create a scatter plot of your data.

NOTE: Clear the STAT area of your calculator by pushing the STAT key, select 4:ClrList, 2nd, L1, enter a comma from the keyboard, 2nd, L2, and then ENTER. The line should look like this prior to pressing ENTER:

ClrList L_1,L_2

- Enter your data into a table by pushing the STAT key and then ENTER. Enter the angle of elevation into L1 and the average distance into L2. Be sure that you have 10 entries in the L1 column and 10 entries in the L2 column before going on to the next step. (Hint: If you have made an error, use the arrow keys to highlight the error and then simply type over the error.)

- Once your data is in the table, create a scatter plot by pushing 2nd and then Y=. Hit ENTER and then move the cursor, using the arrows, to ON and hit ENTER. Now to see the points on the graph, push ZOOM 9. Each data point will now appear on your screen. (Hint: How does this graph differ from the previous graph? Does the data appear to be linear or parabolic?)

- Now we will use the calculator to find the line of best fit of the data. In order to determine the best fit for our data, we will need to go to the Catalog by pushing 2nd and then 0. Scroll down using the arrow keys until you see Diagnostic On. Position the cursor beside it and hit ENTER. Then hit ENTER again and your calculator should say Done.

    - Let's assume that the function is linear: Push the STAT key then use the right arrow to highlight CALC. Push the number 4 then Enter. The calculator will give the line of best fit in the form *ax+b*, where *a* is the slope and *b* is the *y*-intercept. Record the given equation on your paper with each coefficient written to the nearest thousandth. Also, record the r value given.

- Let's now assume that the function is quadratic: Push the STAT key then use the right arrow to highlight CALC. Push the number 5 then ENTER. The calculator will give the quadratic regression in the form $ax^2 + bx + c$. Record the given equation on your paper with each coefficient written to the nearest thousandth. Also, record the R^2 value given.

  (Hint: The r value or R^2 value gives us a numerical picture of how closely the function fits the data. The closer the value is to 1 (or negative 1 in the case of a negative slope), the better fit of the function to the data set. It is generally accepted that a value of .9 or greater is a good fit.)

- Choose which model best fits your data and enter the function into Y=. Then push the GRAPH key. You should now see the data points and the function graphed.

- Record various observations about the data points and the function. How close are the points to the regression? Are there any points that are very far away? What might have caused the discrepancy?

- You will need to include a graph of your data in your final project. You can either neatly graph the data and regression by hand and take a picture or use a computer generated graphing program, such as GeoGebra. The GeoGebra program can be downloaded for free at http://www.geogebra.org/cms/en/download.

5 Fifth Problem: Analyze the data

- Find the vertex of the quadratic model. Is the vertex a maximum or minimum? What does the vertex represent in this situation? (Hint: The TI graphing calculator can quickly calculate the vertex. With the quadratic function entered into Y=, push 2nd and then TRACE. Select Maximum, number 4. The calculator now says Left Bound on the bottom of the screen. Use the arrows to position the cursor to the left of the maximum. Then hit ENTER. The calculator now says Right Bound. Use the arrows to position the cursor to the right of the maximum. Hit ENTER. The calculator will now say Guess. Hit ENTER.)

- Use your linear model from part 2 to determine how far the rubber band must be stretched to travel the maximum horizontal distance found in part 4 above. Hint: Substitute the horizontal distance, *y*, into the linear model and solve for x.

- Now test your calculation. How close did your rubber band come to the maximum horizontal distance?

Collaboration
Trade rubber bands and equations with a neighboring group. Each group should now have their neighbor's linear equation to model flight distance vs. stretch and a quadratic equation to model flight distance vs. angle of elevation. The neighboring group will position a small action figure on the ground within range of the rubber band shooter. The goal is use the two equations to hit the action figure in the least number of tries.

Begin with the linear model. Measure the distance to the action figure in cm. Substitute the measurement in to the linear equation to determine the stretch distance necessary to hit the figure. You many need to adjust slightly. How many tries did it take?

Now try the quadratic model. Substitute the measurement in to the quadratic equation to determine the angle of elevation necessary to hit the figure. Remember that the rubber band should be stretched 5 cm. How many tries did it take?

Which model allowed you to hit the action figure in the least number of tries? With which model was the neighboring group more successful? What factors contribute to the inaccuracy of the model? What could be done to minimize the error?

Conclusions
Your final product will be a multimedia presentation of your experiment photos. You can use the free program Animoto to create a slide show of your photos. The program is available for free at http://animoto.com/create. Another option is Windows Movie Maker or iMovie. Additionally, you will need to present your data tables, graphs, and regression models. You can incorporate the data into your slide show, make a separate poster, or include a typed lab report to go along with your slideshow.

Instructor Notes

Assignment Procedures

Students should produce the following tables and results:

Problem 1
It is helpful to use a package of rubber bands of uniform size and strength. If a group happens to break or lose their rubber band, it can be easily replaced and the experiment can continue. If only varying sizes are available, students will need to begin the experiment again to maintain consistent results.

Problem 2
If a graphing calculator is not available, students can plot their points on any graph and graph a line of best fit by hand. They can then find the equation of the line of best fit by marking any two points on the line and using point-slope form.

Problem 3

It is difficult to hold the ruler at just the right angle and release the rubber band successfully. Encourage the students to take several trial attempts before beginning to collect data.

Recommendations:

- have students work in teams to encourage brainstorming and cooperative learning.
- assign a specific timeline for completion of the project that includes milestone dates.
- provide students feedback as they complete each milestone.
- ensure that each member of student groups has a specific job.

Technology Integration

This project provides abundant opportunities for technology integration, and gives students the chance to research and collaborate using online technology. Several free online resources are suggested in the student instructions:

http://www.geogebra.org/cms/en/download
Geogebra is a mathematical graphics program that can be used to plot test results

http://animoto.com/create.
Animoto can be used to create a slideshow presentation

It may be helpful to go over these or similar programs in the classroom so that all students are comfortable using them.

The following are other examples of free internet resources that can be used to support this project:

http://www.moodle.org
An Open Source Course Management System (CMS), also known as a Learning Management System (LMS) or a Virtual Learning Environment (VLE). Moodle has become very popular among educators around the world as a tool for creating online dynamic websites for their students.

http://www.wikispaces.com/site/for/teachers or http://pbworks.com/content/edu+overview
Lets you create a secure online Wiki workspace in about 60 seconds. Encourage classroom participation with interactive Wiki pages that students can view and edit from any computer. Share class resources and completed student work with parents.

http://www.docs.google.com
Allows students to collaborate in real-time from any computer. Google Docs provides free access and storage for word processing, spreadsheets, presentations, and surveys. This is ideal for group projects.

http://why.openoffice.org/
The leading open-source office software suite for word processing, spreadsheets, presentations, graphics, databases and more. It can read and write files from other

common office software packages like Microsoft Word or Excel and MacWorks. It can be downloaded and used completely free of charge for any purpose.

Rubric

Score	Content	Presentation
4	Your project appropriately answers each of the problems. Completed data tables, graphs, and equations are included. Evidence of careful data collection is apparent. Data points are tightly grouped around the functions.	Your project contains information presented in a logical and interesting sequence that is easy to follow. Your project is professional looking with graphics and attractive use of color.
3	Your project appropriately answers each of the problems. Completed data tables, graphs, and equations are included. Evidence of fairly careful data collection is apparent. Data points are for the most part grouped around the functions. Minor errors may be noted.	Your project contains information presented in a logical sequence that is easy to follow. Your project is neat with graphics and attractive use of color.
2	Your project attempts to answer each of the problems. Partially completed data tables, graphs, and equations are included. Evidence of inaccurate data collection is apparent. Data points are not tightly grouped around the functions. Errors and inaccuracies may be noted.	Your project is hard to follow because the material is presented in a manner that jumps around between unconnected topics. Your project contains low quality graphics and colors that do not add interest to the project.
1	Your project attempts to answer some of the problems. Data tables, graphs, and equations are not complete. Evidence of inaccurate data collection is apparent. Data points are not tightly grouped around the functions. Major errors are noted.	Your project is difficult to understand because there is no sequence of information. Your project is missing graphics and uses little to no color.

Unit 10: Quadratic Functions

axis of symmetry	a line of symmetry for a graph—it divides a figure or graph into halves that are the mirror images of each other
coefficient	a number that multiplies a variable
completing the square	the process of changing a polynomial of the form $x^2 + bx$ into a perfect square trinomial $x^2 + bx + \left(\frac{b}{2}\right)^2$, or $\left(x + \frac{b}{2}\right)^2$
discriminant	the expression b2 – 4ac under the radical in the quadratic formula; the expression can be used to determine the number of real roots the quadratic equation has
function	a kind of relation in which one variable uniquely determines the value of another variable
intercept form of a quadratic equation	written as y = a(x – p)(x – q), where the x-intercepts are p and q
linear function	a function with a constant rate of change and a straight line graph
nonlinear function	a function with a variable rate of change that graphs as a curved line
parabola	a U-shaped graph which is produced by a quadratic equation
perfect square	any of the squares of the integers. Since 12 = 1, 22 = 4, 32 = 9, etc., 1, 4, and 9 are perfect squares
polynomial	a monomial or sum of monomials, like 4x2 + 3x – 10
polynomial functions	a monomial or sum of monomials, like y = 4x2 + 3x – 10
quadratic equation	an equation that can be written in the form ax2 + bx + c = 0 where a ¹ 0. When written as y = ax2 + bx+ c the expression becomes a quadratic function.
quadratic formula	the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; it is used to solve a quadratic equation of the form $ax^2 + bx + c = 0$
quadratic function	a function of the form y = ax2 + bx + c where a is not equal to zero
range	the set of all possible outputs of a function
roots of a quadratic	the x-intercepts of the parabola or the solution of the equation

equation	
standard form of a quadratic equation	written as $y = ax^2 + bx + c$, where x and y are variables and a, b, and c are numbers with a ≠ 0. In the case of a single variable the standard form becomes ax2 + bx + c = 0.
system of equations	a set of two or more equations that share two or more unknowns
trinomial	a three-term polynomial
vertex	the high point or low point of a parabolic function
vertex form of a quadratic equation	when the quadratic equation is a quadratic function, the vertex form is $y = a(x-h)^2 + k$, where x andy are variables and a, h, and k are numbers – the vertex of this parabola has the coordinates (h, k)
x-intercept	the point where a line meets or crosses the x-axis
Zero Product Property	states that if ab = 0, then either a = 0 or b = 0, or both a and b are 0

NROC Algebra 1--An Open Course
Unit 10: Quadratic Functions
Mapped to Common Core State Standards, Mathematics

Unit 10, Lesson 1, Topic 1: Graphing Quadratic Functions

Grade: **9-12** - Adopted **2010**

STRAND / DOMAIN	**CC.A.**	Algebra
CATEGORY / CLUSTER	**A-CED.**	Creating Equations
STANDARD		Create equations that describe numbers or relationships.
EXPECTATION	**A-CED.2.**	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
STRAND / DOMAIN	**CC.A.**	Algebra
CATEGORY / CLUSTER	**A-REI.**	Reasoning with Equations and Inequalities
STANDARD		Understand solving equations as a process of reasoning and explain the reasoning.
EXPECTATION	**A-REI.1.**	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
STRAND / DOMAIN	**CC.A.**	Algebra
CATEGORY / CLUSTER	**A-REI.**	Reasoning with Equations and Inequalities
STANDARD		Solve equations and inequalities in one variable.
EXPECTATION	**A-REI.4.**	Solve quadratic equations in one variable.
GRADE EXPECTATION	**A-REI.4.b.**	Solve quadratic equations by inspection (e.g., for x^2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a plus-minus bi for real numbers a and b.
STRAND / DOMAIN	**CC.A.**	Algebra
CATEGORY / CLUSTER	**A-REI.**	Reasoning with Equations and Inequalities
STANDARD		Represent and solve equations and inequalities graphically.
EXPECTATION	**A-REI.10.**	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
STRAND / DOMAIN	**CC.F.**	Functions
CATEGORY / CLUSTER	**F-IF.**	Interpreting Functions
STANDARD		Interpret functions that arise in applications in terms of the context.

EXPECTATION	**F-IF.4.**	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
STRAND / DOMAIN	**CC.F.**	Functions
CATEGORY / CLUSTER	**F-IF.**	Interpreting Functions
STANDARD		Analyze functions using different representations.
EXPECTATION	**F-IF.7.**	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
GRADE EXPECTATION	**F-IF.7.a.**	Graph linear and quadratic functions and show intercepts, maxima, and minima.
STRAND / DOMAIN	**CC.F.**	Functions
CATEGORY / CLUSTER	**F-BF.**	Building Functions
STANDARD		Build new functions from existing functions.
EXPECTATION	**F-BF.4.**	Find inverse functions.
GRADE EXPECTATION	**F-BF.4.a.**	Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) =2 x^3 for x > 0 or f(x) = (x+1)/(x-1) for x not equal to 1.

Unit 10, Lesson 1, Topic 2: Solving Quadratic Equations by Completing the Square

Grade: **9-12** - Adopted **2010**

STRAND / DOMAIN	**CC.A.**	Algebra
CATEGORY / CLUSTER	**A-SSE.**	Seeing Structure in Expressions
STANDARD		Write expressions in equivalent forms to solve problems.
EXPECTATION	**A-SSE.3.**	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
GRADE EXPECTATION	**A-SSE.3.b.**	Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
STRAND / DOMAIN	**CC.A.**	Algebra
CATEGORY / CLUSTER	**A-REI.**	Reasoning with Equations and Inequalities
STANDARD		Understand solving equations as a process of reasoning and explain the reasoning.
EXPECTATION	**A-REI.1.**	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
STRAND / DOMAIN	**CC.A.**	Algebra
CATEGORY / CLUSTER	**A-REI.**	Reasoning with Equations and Inequalities

STANDARD		Solve equations and inequalities in one variable.
EXPECTATION	**A-REI.4.**	Solve quadratic equations in one variable.
GRADE EXPECTATION	**A-REI.4.a.**	Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
GRADE EXPECTATION	**A-REI.4.b.**	Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a plus-minus bi for real numbers a and b.
STRAND / DOMAIN	**CC.F.**	Functions
CATEGORY / CLUSTER	**F-IF.**	Interpreting Functions
STANDARD		Analyze functions using different representations.
EXPECTATION	**F-IF.8.**	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
GRADE EXPECTATION	**F-IF.8.a.**	Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
STRAND / DOMAIN	**CC.F.**	Functions
CATEGORY / CLUSTER	**F-BF.**	Building Functions
STANDARD		Build new functions from existing functions.
EXPECTATION	**F-BF.4.**	Find inverse functions.
GRADE EXPECTATION	**F-BF.4.a.**	Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) =2 x^3$ for x > 0 or f(x) = (x+1)/(x-1) for x not equal to 1.

Unit 10, Lesson 1, Topic 3: Solving Quadratic Equations Using the Quadratic Formula

Grade: **9-12** - Adopted **2010**

STRAND / DOMAIN	**CC.A.**	Algebra
CATEGORY / CLUSTER	**A-CED.**	Creating Equations
STANDARD		Create equations that describe numbers or relationships.
EXPECTATION	**A-CED.2.**	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
STRAND / DOMAIN	**CC.A.**	Algebra
CATEGORY / CLUSTER	**A-REI.**	Reasoning with Equations and Inequalities
STANDARD		Understand solving equations as a process of reasoning and explain the reasoning.

EXPECTATION	**A-REI.1.**	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
STRAND / DOMAIN	**CC.A.**	Algebra
CATEGORY / CLUSTER	**A-REI.**	Reasoning with Equations and Inequalities
STANDARD		Solve equations and inequalities in one variable.
EXPECTATION	**A-REI.4.**	Solve quadratic equations in one variable.
GRADE EXPECTATION	**A-REI.4.a.**	Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x - p)^2 = q that has the same solutions. Derive the quadratic formula from this form.
GRADE EXPECTATION	**A-REI.4.b.**	Solve quadratic equations by inspection (e.g., for x^2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a plus-minus bi for real numbers a and b.
STRAND / DOMAIN	**CC.A.**	Algebra
CATEGORY / CLUSTER	**A-REI.**	Reasoning with Equations and Inequalities
STANDARD		Represent and solve equations and inequalities graphically.
EXPECTATION	**A-REI.10.**	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
STRAND / DOMAIN	**CC.F.**	Functions
CATEGORY / CLUSTER	**F-IF.**	Interpreting Functions
STANDARD		Interpret functions that arise in applications in terms of the context.
EXPECTATION	**F-IF.4.**	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
STRAND / DOMAIN	**CC.F.**	Functions
CATEGORY / CLUSTER	**F-IF.**	Interpreting Functions
STANDARD		Analyze functions using different representations.
EXPECTATION	**F-IF.7.**	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
GRADE EXPECTATION	**F-IF.7.a.**	Graph linear and quadratic functions and show intercepts, maxima, and minima.
STRAND / DOMAIN	**CC.F.**	Functions
CATEGORY / CLUSTER	**F-BF.**	Building Functions

STANDARD		Build new functions from existing functions.
EXPECTATION	**F-BF.4.**	Find inverse functions.
GRADE EXPECTATION	**F-BF.4.a.**	Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) =2 x^3 for x > 0 or f(x) = (x+1)/(x-1) for x not equal to 1.

Unit 10, Lesson 2, Topic 1: Applications of Quadratic Functions

Grade: **7** - Adopted **2010**

STRAND / DOMAIN	**CC.7.EE.**	Expressions and Equations
CATEGORY / CLUSTER		Use properties of operations to generate equivalent expressions.
STANDARD	**7.EE.2.**	Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."

Grade: **9-12** - Adopted **2010**

STRAND / DOMAIN	**CC.A.**	Algebra
CATEGORY / CLUSTER	**A-CED.**	Creating Equations
STANDARD		Create equations that describe numbers or relationships.
EXPECTATION	**A-CED.1.**	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
EXPECTATION	**A-CED.2.**	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
EXPECTATION	**A-CED.3.**	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.
STRAND / DOMAIN	**CC.A.**	Algebra
CATEGORY / CLUSTER	**A-REI.**	Reasoning with Equations and Inequalities
STANDARD		Understand solving equations as a process of reasoning and explain the reasoning.
EXPECTATION	**A-REI.1.**	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
STRAND / DOMAIN	**CC.A.**	Algebra
CATEGORY / CLUSTER	**A-REI.**	Reasoning with Equations and Inequalities
STANDARD		Solve equations and inequalities in one variable.
EXPECTATION	**A-REI.4.**	Solve quadratic equations in one variable.

GRADE EXPECTATION	**A-REI.4.b.**	Solve quadratic equations by inspection (e.g., for x^2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a plus-minus bi for real numbers a and b.
STRAND / DOMAIN	**CC.F.**	Functions
CATEGORY / CLUSTER	**F-IF.**	Interpreting Functions
STANDARD		Interpret functions that arise in applications in terms of the context.
EXPECTATION	**F-IF.4.**	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
STRAND / DOMAIN	**CC.F.**	Functions
CATEGORY / CLUSTER	**F-IF.**	Interpreting Functions
STANDARD		Analyze functions using different representations.
EXPECTATION	**F-IF.7.**	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
GRADE EXPECTATION	**F-IF.7.a.**	Graph linear and quadratic functions and show intercepts, maxima, and minima.
STRAND / DOMAIN	**CC.F.**	Functions
CATEGORY / CLUSTER	**F-BF.**	Building Functions
STANDARD		Build a function that models a relationship between two quantities.
EXPECTATION	**F-BF.1.**	Write a function that describes a relationship between two quantities.
GRADE EXPECTATION	**F-BF.1.a.**	Determine an explicit expression, a recursive process, or steps for calculation from a context.
STRAND / DOMAIN	**CC.F.**	Functions
CATEGORY / CLUSTER	**F-BF.**	Building Functions
STANDARD		Build new functions from existing functions.
EXPECTATION	**F-BF.4.**	Find inverse functions.
GRADE EXPECTATION	**F-BF.4.a.**	Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) =2 x^3 for x > 0 or f(x) = (x+1)/(x-1) for x not equal to 1.

Unit 10, Lesson 2, Topic 2: Systems of Non-Linear Equations

Grade: **9-12** - Adopted **2010**

STRAND / DOMAIN	**CC.A.**	Algebra
CATEGORY / CLUSTER	**A-REI.**	Reasoning with Equations and Inequalities

STANDARD		Understand solving equations as a process of reasoning and explain the reasoning.
EXPECTATION	**A-REI.1.**	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
STRAND / DOMAIN	**CC.A.**	Algebra
CATEGORY / CLUSTER	**A-REI.**	Reasoning with Equations and Inequalities
STANDARD		Solve systems of equations.
EXPECTATION	**A-REI.7.**	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line y = -3x and the circle x^2 + y^2 = 3.
STRAND / DOMAIN	**CC.F.**	Functions
CATEGORY / CLUSTER	**F-BF.**	Building Functions
STANDARD		Build new functions from existing functions.
EXPECTATION	**F-BF.4.**	Find inverse functions.
GRADE EXPECTATION	**F-BF.4.a.**	Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) =2 x^3 for x > 0 or f(x) = (x+1)/(x-1) for x not equal to 1.